TIME AVERAGE SCANNING DIGITAL HOLOGRAPHY

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Abstract

This paper presents a novel method for amplitude distribution measurement of harmonically oscillating objects called time average scanning digital holography (TASDH). Reconstructed intensity image from time averaged digital hologram has the form of a set of fringes that follow a zero-order Bessel function of the first kind. When the phase of light beam in the interferometer is modulated, the bright zero-order fringe shifts with respect to the modulation depth. The method is based on continuous shift of the zero-order fringe over the object surface and the value of vibration amplitude is evaluated independently in every single pixel. An advantageous feature of the proposed technique with respect to other digital holographic vibrometry methods is the ability to measure amplitudes of vibration without the risk of interference fringe order, or 2π, errors. On that account, the method allows to measure high slopes amplitudes distribution or discontinuous/partially shaded objects. The correctness of the method was experimentally verified by measuring the bending piezo actuator.

Keywords

Digital holography; Time-average holography; Scanning holography; Vibrations; Phase modulation.

Introduction

Vibrometry is an integral part of various technical and scientific disciplines. There is a broad portfolio of vibration measurement devices, which are usually based on the Doppler phenomenon [1], correlation analysis [2], speckle ESPI (Electronic Speckle Pattern Interferometry) [3], and others. These methods are mostly single-point methods or their measuring capabilities in terms of maximal amplitudes or frequency range are limited. Thus, an extra care must be taken to parameters of a device when choosing one for a certain application. Another suitable tool for analyzing vibrations in the whole surface are digital holography based methods.

The holographic approach to measurement of vibration depends mainly on the frequency of the measured phenomena. For very low frequencies (much lower than FPS of the digital camera) it is possible to use "classical" digital holographic interferometry [4]. The first hologram is captured at steady-state without object oscillation while other holograms are captured at different phases of the movements. The measured interference phase between the reference and the hologram at a certain phase of the movement is proportional to vibration amplitude of the object. This approach can be used unless the hologram within the exposure time is blurred by the object movement. High-speed cameras allowing very short exposure time together with the use of high-power laser can push the limits [5]. Similar approach for high frequencies is called the stroboscopic method [6]. The method requires using of a coherent pulse laser synchronized with digital camera. Short light pulses "freeze" the oscillating movement of the object at certain phase of the movement and therefore the hologram is not blurred. The interference phase between steady-state hologram and hologram at certain phase of the movement is measured and converted to vibration amplitude and phase.

The above mentioned methods were conditioned by short exposure time when compared to the period of the object oscillation. However, much longer exposure time than the period of the oscillation leads to so called time average digital holography (TADH). The intensity image reconstructed from such digital hologram exhibits a system of interference fringes described by Bessel function, which map contours of constant vibration amplitude. On that account high frequencies can be holographically measured without need of any special equipment. A lot of work has been done in order to modify the method for very low [7-9] or large [10, 11] amplitudes.

One of the most challenging tasks in TADH is to qualitatively determine the amplitude distribution from the fringe pattern modulated by Bessel function. Numerical analysis of cosine fringes (exhibiting in other interferometric techniques) by phase-shifting techniques [12] allows the determination of interference phases even between the fringe intensity maxima and minima with high accuracy. Phase-shifting technique was also developed for TADH employing phase modulation by mirror mounted on piezoelectric transducer [13] or acusto-optical modulator [14] in order to shift the Bessel fringes similarly to phase stepping in case of cosine fringes. This resulted in relative quantitative measurement of amplitudes of vibrations independently in every single pixel. Such methods, however, have some limitations. On the one hand, it is necessary to correct the result due to the difference between cosine and Bessel functions. Further, a spatial unwrapping algorithm is needed to demodulate the wrapped phase field and determine the "zero point" (oscillation node) from where other pixels are relatively measured. Naturally, the methods fail when it comes to vibration modes with high slopes leading to very dense fringe patterns. Moreover, for the correct amplitude distribution evaluation the nodal line is supposed to be within the field of view and simultaneously the FOV should be a continuous area without shading effects, etc.

We propose scanning time average scanning digital holography (TASDH) tackling the aforementioned drawbacks. It is also based on the phase modulation of the reference or object wave. The alter phase is now continuously scanned and using non-harmonic properties of zero-order first kind Bessel function we can determine the amplitude of vibration absolutely in every single pixel. Therefore, no spatial unwrapping or Cosine-Bessel correction is required. Although the principle is different, TASDH can be qualitatively compared to coherence scanning interferometry or white light. In comparison to phase-shifting methods, TASDH needs more data to be acquired, which is no problem for today's computers. This paper introduces the basic principle of TASDH and its experimental verification.

1 Aim of Research

At the highest level, the research focuses on environmental noise management. More specifically, the focus is on sounds that people prefer to hear, and the acoustic environment is considered as a resource. The absolute elimination of all sounds is not natural, and the strategy of general noise suppression has been overcome by the possibilities of new technologies. Such a novel approach within soundscape planning allows elimination of just particular unwanted sounds and purifies the acoustic environment within the controlled space.

Implementation of the soundscape planning principles requires the research and development of efficient tools for a delicate control of sound wave propagation in an open space. Within this research project, our attention is focused on so called acoustic metasurfaces (AMS). Such research must cover several areas. Besides the development of AMS, it is mainly methods for measuring of specific acoustic impedance that control the transmission of sound waves through interfaces between two different media. Holographic methods allow fast and precise measurements of vibration amplitudes over the whole inspected area with high lateral resolution and are therefore one of the pillars of the whole research aim.

2 Time Average Scanning Digital Holography

2.1 Digital Holography

Digital holography (DH) involves the recording and reconstruction of optical waves. The recording is based on the superposition of a wave scattered from the object surface (called the object wave) *O* with a known reference wave *R* and recording their interference pattern. This interference pattern is called a digital hologram and can be described by the interference formula [4]:

$$H = \left|O + R\right|^2 = \left|R\right|^2 + \left|O\right|^2 + O^* R + OR^* . \qquad (1)$$

The hologram h*H* carries information about the intensity and phase of the waves and is stored in computer as an array of numbersU_o.

The reconstruction retrieves the information of the amplitude and phase from the digital hologram. The digital hologram *H* is multiplied with a numerical representation of conjugated reference wave R^* Ur^*, which results in a complex wave field in the hologram plane (with coordinates notation ξ, η). This wave field is then numerically propagated in a free space according to the laws of diffraction and the resulting complex field is calculated in a certain reconstruction distance *d* called the image plane (coordinates notations *x*, *y*). The free space propagation can be computed by Fresnel approximation [4]:

$$U(x, y) = \Im^{-1}\left\{H(\xi,\eta) R^*(\xi,\eta) \exp\left[-\frac{j\pi}{\lambda d}\left(\xi^2 + \eta^2\right)\right]\right\}, \qquad (2)$$

where $\Im^{-1}$ F^{-1} denotes the inverse discrete Fourier transform and λ is the wavelength of the laser. Both hologram and image plane are sampled by $N \times M$ pixels. Pixel extension in the hologram plane $\Delta\xi \times \Delta\eta$ is naturally defined by real pixel extension of the sensor. The image plane pixel dimensions Δx , Δy are given by parameters of the reconstruction [4]:

$$x = \frac{\lambda d}{N\Delta\xi} \quad \text{and} \quad \Delta y = \frac{\lambda d}{M\Delta\eta} . \qquad (3)$$

The result of the reconstruction process $U(n\Delta x, m\Delta y)$is a complex wave field in the image plane $U(x, y)$ from which the intensity $I(n\Delta x, m\Delta y)$and phase distributions can be computed as [4]:

$$I(x, y) = \left|U(x, y)\right| \quad \text{and} \quad \varphi(x, y) = \arg\left(U(x, y)\right). \qquad (4)$$

2.2 Principle of the Method

Holographical recording of a harmonically oscillating object with an exposure time much longer when compared to the period of the object vibrations results in so called time average holography. Considering amplitude distribution $\vec{d}(x,y,t)=\vec{d}(x,y)\sin(\omega t)$ oscillating in time t, where $\vec{d}$ is the amplitude of vibration and ω stands for the angular frequency, the intensity of the reconstructed image is proportional to magnitude of the first kind zero-order Bessel function J_0 [4]:

$$I(x,y)=\left|J_0(\varphi(x,y))\right|. \tag{5}$$

In equation (5) the argument of the Bessel function (further called interference phase) $\varphi(x,y)=\vec{d}(x,y)\vec{e}(x,y)$ is proportional to amplitude of the vibration and sensitivity vector $\vec{e}$, which is given by the geometrical properties of the holographic arrangement. We can assume only out-of-plane vibrations $\vec{d}=[0,0,D]$ and perpendicular illumination and observation of the object $\vec{e}=2\pi/\lambda[0,0,2]$ yielding in [4]:

$$\varphi(x,y)=D(x,y)\,4\pi/\lambda. \tag{6}$$

The TASDH requires the phase modulation of the reference R or the object O beam with the same frequency ω as the oscillating object and with a modulation depth Ω:

$$O(t)=O\exp(jB\sin(\omega t)). \tag{7}$$

Putting the phase modulated object wave (7) into interference equation (1) modifies equation (5) into:

$$I(x,y,\Omega)=\left|J_0(\varphi(x,y)-\Omega)\right|. \tag{8}$$

Now, the reconstructed intensity image involves the modulation depth Ω. The loci of bright zero fringes appear where

$$\varphi(x,y)=\Omega \tag{9}$$

and can be unambiguously determined. Scanning of the modulation depth Ω provides an intensity signal for each image pixel as a function of the modulation depth (6). The modulation depth is not exactly known since it is defined by the transfer function between user controllable electronic device and the real phase modulation of the light beam. Therefore the interference phase φ is computed as the envelope shift between steady-state ($\varphi=0,\Omega=0$) with intensity:

$$I(x,y,0)=\left|J_0(0)\right| \tag{10}$$

and the oscillating state defined in equation (6). Although there are several strategies how to search for the envelope shift, due to noise resistance we decided for cross-correlation (operator denoted by $*$) based searching using formula:

$$C(\hat{\Omega})=I_0*I=\int I_0(\Omega)I(\Omega+\hat{\Omega})d\Omega, \tag{11}$$

where the lag of the cross-correlation function $\hat{\Omega}$ has the physical meaning of the modulation depth Ω. The maximum of the cross-correlation function $C(\hat{\Omega})$ indicates the lag where the signals are best aligned and $\hat{\Omega} \approx \varphi$. The interference phase can be therefore computed as:

$$\varphi = \arg\max(I_0 * I)\,. \tag{12}$$

Hence, the out-of-plane amplitude $D(x,y)$ is computed as simple inversion of (6):

$$D(x,y) = \varphi(x,y)\,\lambda/4\pi\,. \tag{13}$$

3 Experiment and Results

The experimental arrangement used for verification of TASDH principles is introduced in Fig. 1. The Nd:YAG laser emits coherent light with the wavelength of 532 nm that is split into the reference and the object arm. Both arms of the holographic interferometer are spatially filtered and imping the sensor of a digital camera having 2048 × 2048 pixels each of size 3.45 µm × 3.45 µm. Superposition of the reference and the object wave generates an interference structure that is captured by the digital camera and creates a digital hologram. The phase of the object wave is on the one hand modulated by the object vibrations and on the other hand by the oscillating mirror mounted on an electronically controlled piezoelectric transducer (PZT). Naturally, the PZT mirror can also be placed in the reference arm.

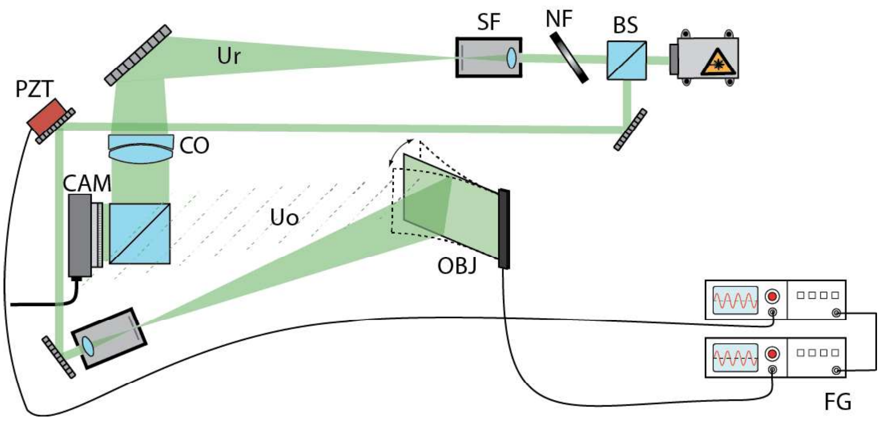

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Fig. 1: *Principal scheme of a holographic interferometer for TASDH employing components: BS – beam splitter, NF - neutral density filter, SF – spatial filter, CO – collimating objective, OBJ – object, FG – arbitrary waveform generator, CAM – digital camera, M – mirror, PZT – piezoelectric transducer, Ur denotes reference wave while Uo stands for object wave.*

The investigated object is a bending piezoelectric actuator driven by a two channel waveform generator. One channel of the waveform generator sets the frequency and amplitude of the harmonic object vibrations, while the second channel controls the PZT mirror.

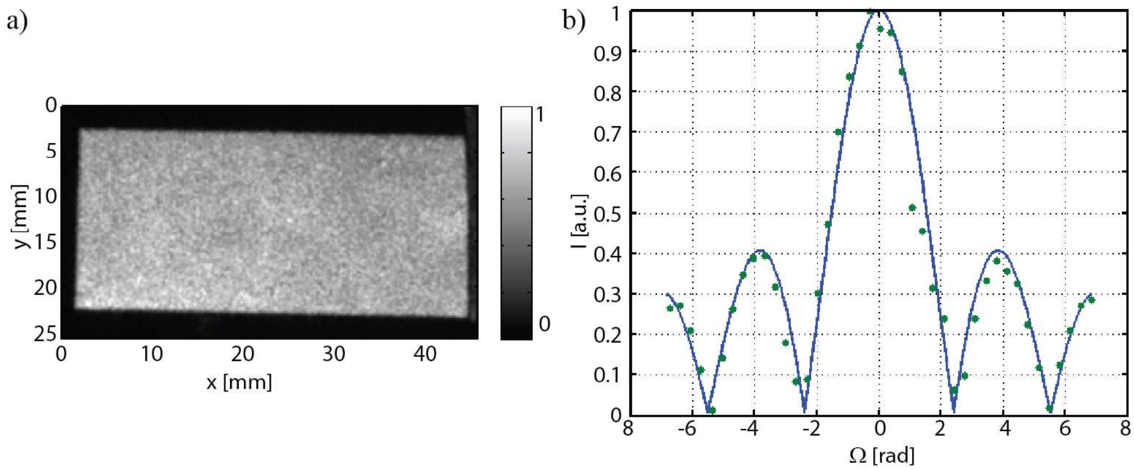

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Fig. 2: *a) Reconstructed intensity image of steady-state object with no phase modulation $\Omega = 0$; b) measured (green) and fitted (blue) intensity values at x = 25 mm and y = 13 mm as a function of the modulation depth*

In the first step, the object is in steady-state and the PZT is oscillating with a frequency of 700 Hz. The driven amplitude of PZT was scanned in the range from -20 VPP (peak-to-peak) to 20 VPP with the step of 1 VPP. The reconstructed intensity image of steady-state object with no phase modulation $\Omega = 0$ is shown in Fig. 2a. As expected, the intensity at any object point (in our case x = 25 mm, y = 13 mm) is a function of the phase modulation depth and in every pixel follows equation (8) with $\varphi = 0$, see Fig. 2b.

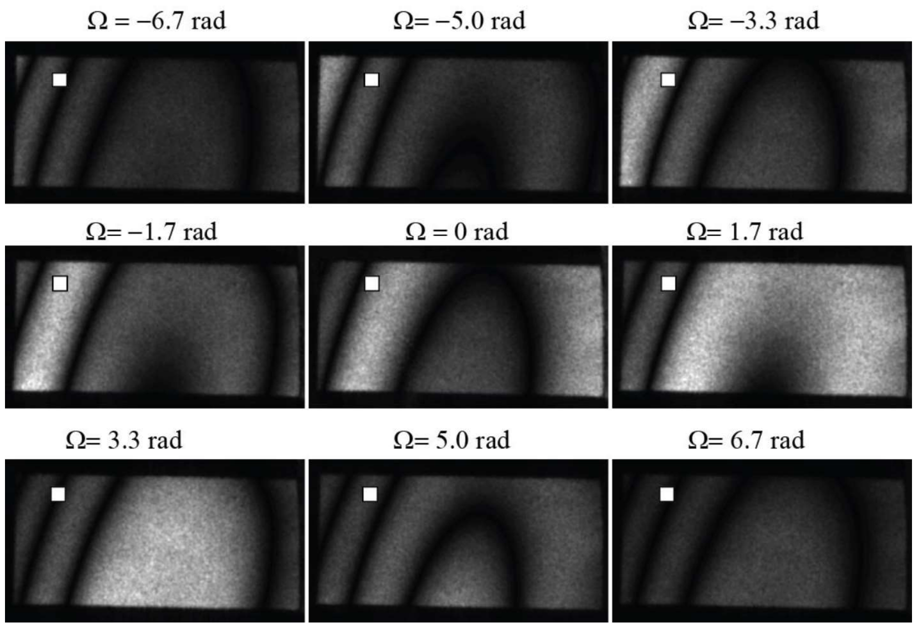

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Fig. 3: *Sequence of intensity images of oscillating object with different phase modulation depths. The white square denotes pixel used for analysis in Fig. 4.*

The green markers are measured values and the blue line is fitted magnitude of Bessel function. The results of the Bessel function fitting provide also the transfer function between the electronic output signal (PZT driven amplitude) from the waveform generator and the phase modulation depth of the light wave: 1 VPP ~ 0.34 rad. The measured or fitted result can be used as reference for the following measurement.

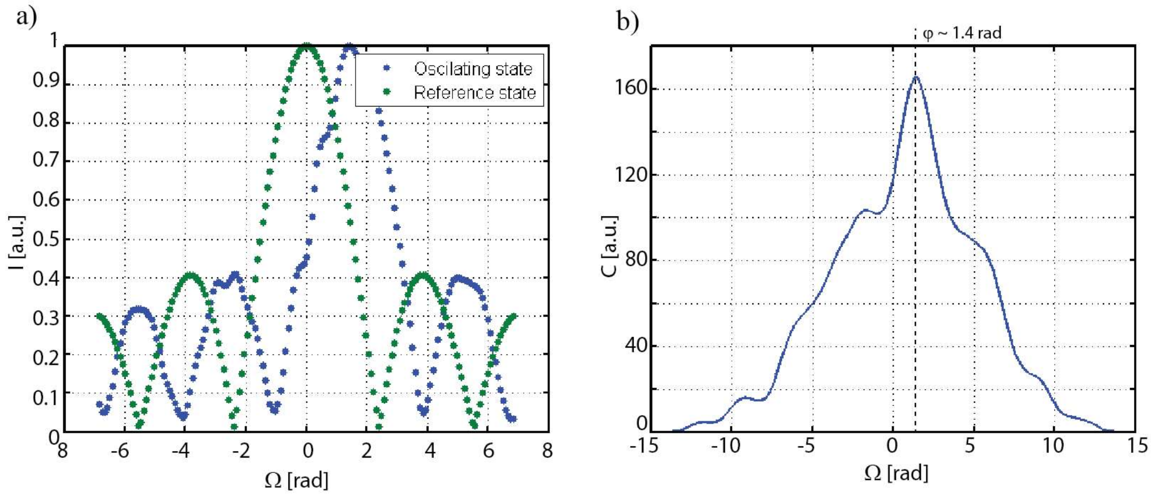

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Fig. 4: *Interference phase evaluation in the "white square" pixel (see Fig. 3): a) plot of steady-state (reference) Bessel function and measured intensity values as a function of phase modulation depth that form shifted Bessel function; b) cross-correlation of green and blue functions in a) with highlighted lag corresponding to the value of interference phase.*

In the second step, we set the object frequency to 700 Hz with 1 VPP driven amplitude. The frequency of the PZT remains 700 Hz, the start phase of the PZT is aligned with the object vibration phase and the modulation depth (driven amplitude) is scanned from -20 VPP to +20 VPP with the step of 1 VPP. The sequence of some intensity images for different phase modulation in Fig. 3 illustrates how the bright fringe shifts over the object surface. Further, one pixel (at position of the white square) is chosen for detailed analysis. Green markers in Fig. 4a plots reference (steady-state) Bessel function analogously to Fig. 2b and the measured values in the "white square" pixel - blue markers - are represented by the shifted Bessel function. Maximum of cross-correlation function between the green and blue functions (plotted in Fig. 4b) indicates the lag corresponding to the interference phase $\hat{\Omega} \approx \varphi$ as discussed in equation (12). This procedure is realized independently in every single pixel and using formula (13) the interference phase is converted to amplitude distribution, see Fig. 5.

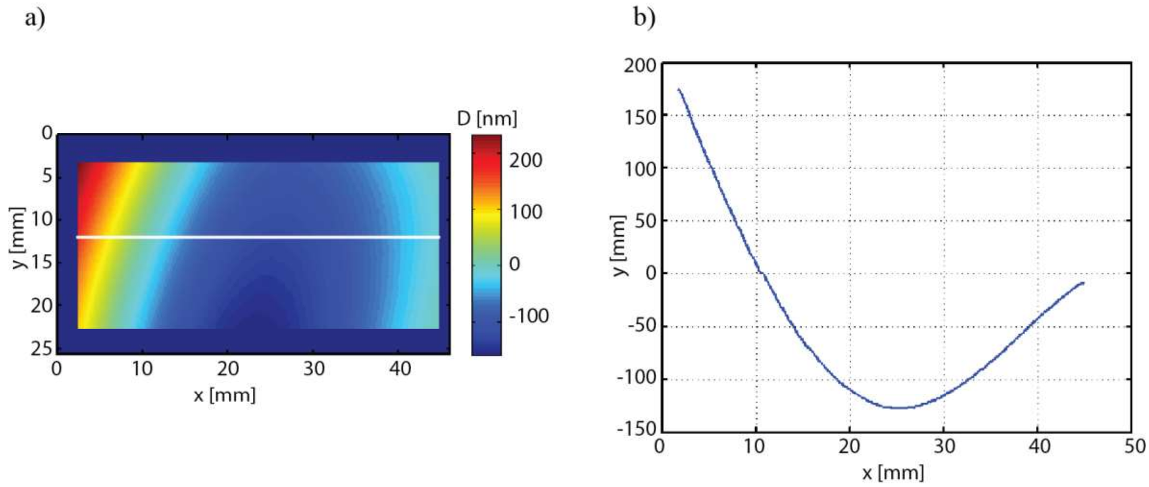

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Fig. 5: *Amplitude distribution measured by TASDH: a) false color image, where different colors represent different values of out-of-plane vibration amplitudes; b) vibration amplitude values along white line in a).*

To verify the correctness of the result, we substitute the measured interference phase φ into equation (5). The resulting intensity map (Fig. 6a) should correspond to the reconstructed intensity image with no modulation (Fig. 6b). Both images are in very good agreement.

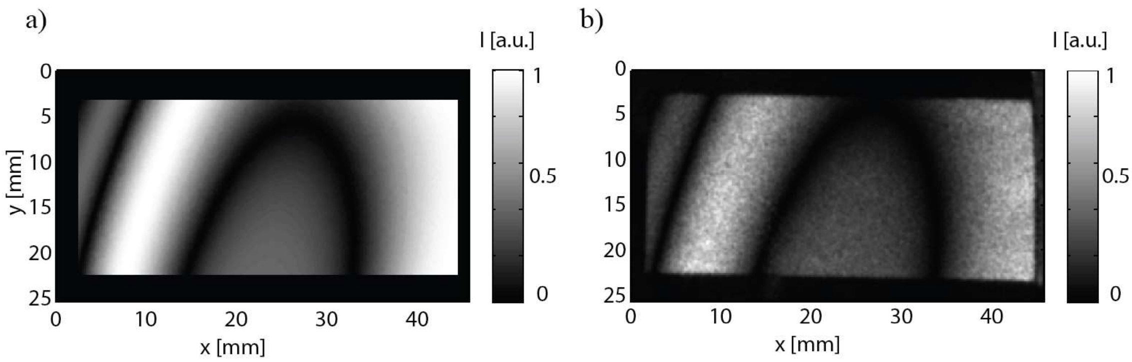

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Fig. 6: *Verification of the results: a) intensity image computed from evaluated interference phase by TASDH; b) measured intensity image; the both images are in very good agreement.*

Conclusion

This paper presents basic principles and experimental verification of novel time average scanning digital holography (TASDH). This method is applied to the measurement of harmonic vibration amplitudes. The reconstructed intensity image from the time averaged digital hologram consists a fringe pattern structure (described by Bessel function) reflecting contours of vibration amplitudes. The principle of the TASDH method is based on phase modulation of light beam in the object or reference arm of the holographic interferometer. As the phase modulation depth varies, the fringes in the reconstructed intensity image continuously shift over the object surface and the brightest zero order fringe can be easily located. The interference phase value corresponding to this zero order fringe is ambiguously known and therefore can be easily linked to the certain surface point. This procedure is done independently in every single pixel and thus no spatial unwrapping is needed. An advantageous feature of the TASDH technique with respect to other quantification methods within TADH is the ability to measure amplitudes of vibration without the risk of interference fringe order, or 2π, errors. On that account the method allows to measure high slopes amplitudes distribution or discontinuous/partially shaded objects. The correctness of the method was experimentally verified by measuring the bending piezo actuator.

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ČASOVĚ STŘEDOVANÁ SKENOVACÍ DIGITÁLNÍ HOLOGRAFIE

Tento článek představuje novou metodu pro měření amplitud harmonických oscilací nazvanou časově středovaná skenovací digitální holografie. Rekonstruovaný intenzitní obraz z časově středovaného digitálního hologramu vytváří interferenční strukturu, která je popsána Besselovou funkcí prvního typu nultého řádu. Je-li fáze laserového svazku v interferometru modulována, interferenční proužek odpovídající nultému řádu se v závislosti na hloubce modulace posune. Metoda je založena na skenování proužku nultého řádu přes celý povrch objektu. Amplituda vibrací je nezávisle a absolutně vyhodnocena v každém pixelu. Výhodou navrhované techniky ve srovnání s dalšími metodami digitální holografické vibrometrie je schopnost měřit amplitudu vibrací bez rizika správnosti určení interferenčního řádu. Z tohoto důvodu metoda umožňuje měření amplitud vibrací s velkou strmostí či nespojitých / částečně zastíněných objektů. Správnost metody byla experimentálně ověřena měřením ohybového piezoelektrického aktuátoru.

ZEITDURCHSCHNITT DER SCAN-DIGITAL-HOLOGRAPHIE

Diese Arbeit stellt ein neuartiges Verfahren zur Amplitudenverteilung harmonisch oszillierender Objekte vor, das als zeitgemittelte digitale Holographie bezeichnet wird. Das rekonstruierte Bild des zeitgemittelten digitalen Hologramms hat die Form eines Satzes von Streifen, die einer Bessel-Funktion nullter Ordnung des ersten Typs folgen. Wenn die Phase des Lichtstrahls in dem Interferometer moduliert wird, verschiebt sich der helle Streifen nullter Ordnung in Bezug auf die Modulationstiefe. Das Verfahren basiert auf der kontinuierlichen Verschiebung des Streifens nullter Ordnung über die Objektoberfläche und der Wert der Schwingungsamplitude wird unabhängig in jedem Pixel bewertet. Ein vorteilhaftes Merkmal der vorgeschlagenen Technik in Bezug auf andere digitale holographische Vibrometrieverfahren ist die Fähigkeit, die Schwingungsamplitude ohne Risiko von Interferenzstreifenordnungen oder 2π-Fehlern zu messen. Aus diesem Grund erlaubt das Verfahren die Messung großer Amplitudenverteilungen oder diskontinuierlicher / teilweise schattierter Objekte. Die Richtigkeit der Methode wurde experimentell durch die Messung des Biegepiezoaktors überprüft.

CZASOWO WYŚRODKOWANA CYFROWA HOLOGRAFIA SKANINGOWA

W artykule przedstawiono nowatorską metodę pomiaru amplitud oscylacji harmonicznych nazwaną czasowo wyśrodkowaną cyfrową holografią skaningową. Obraz odtworzony z czasowo wyśrodkowanego cyfrowego hologramu tworzy strukturę interferencyjną, opisaną funkcją Bessela pierwszego rodzaju zerowego rzędu. Jeżeli faza wiązki laserowej w interferometrze jest modulowana, pasek interferencyjny odpowiadający rzędowi zerowemu w zależności od głębokości modulacji przesuwa się. Metoda oparta jest na skanowaniu paska rzędu zerowego na całej powierzchni obiektu. Amplitudę drgań oceniano niezależnie i absolutnie dla każdego piksela. Zaletą proponowanej techniki w porównaniu z innymi metodami cyfrowej wibrometrii holograficznej jest zdolność pomiaru amplitudy drgań bez ryzyka dla prawidłowego określenia rzędu interferencyjnego. Z tego powodu metoda pozwala na pomiar amplitud wibracji o dużej stromości lub niespójnych/częściowo zacenionych obiektów. Prawidłowość metody została zweryfikowana eksperymentalnie poprzez pomiar giętego piezoelektrycznego aktuatora.