GRAPH-BASED ANALYSIS OF PLANETARY GEARS

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Abstract

In this paper, the graph-based models of planetary gears are discussed. The considered methods are as follows: linear (Hsu's) graph, contour (Marghitu's) graphs and bondgraphs. The original rules for the use of graph based methods were modified by the authors making them more adequate and more useful. The selected planetary gear was analyzed to show a course of calculations for the proposed methods in comparison with the traditional Willis method. The advantages of the graph based method consist in submission of an algebraic structure of a gear – which allows for an application of some AI approaches. Bondgraphs models of a subgear were presented. It additionally allows for algorithmic generation not only kinematical equations but also for relationships according to forces and accelerations or power.

Introduction

Planetary gears are modeled by means of many different approaches. However, just recently, their graph-based models [1-10] are intensively developed. The following graphs were used for this task: bondgraphs [1, 9], linear graphs [2, 3, 8] and contour graphs [2, 3, 5] as well as some others [9, 10].

In this paper, the aforementioned graphs were used for kinematical analysis of aselected planetary gear. The applied methods were utilized and compared. The aim of this paper is to show the advantages of the graph-based method i.e. algorithmic approach.

The additional benefits of modeling of gears via graphs consist in submission of algebraic encoding of the structure of a planetary gear [10]. It – in turn – allows for an application of Artificial Intelligence (AI) approaches and methods [4,6,7,8] (e.g.: graph grammar and evolutionary algorithms). Due to this, a designer has a powerful tool for supporting several design tasks at the conceptual stage of the design. In general, such activities in graph modeling have been performed as for example: synthesis of planetary gears [3] and enumeration of their kinematic structures [8] which allows for creation of atlases of all possible design solutions.

We calculated the ratio of the considered selected planetary gear by means of the traditional Willis method as well as graph based methods. In case of bondgraphs, only the introductory stage of modeling is shown here. In fact, this method is especially powerful because it enables simulation [1], but it exceeds the assumed scope of this paper.

1 A selected planetary gear

The functional scheme of the considered selected planetary gear is presented in *Fig.1*.

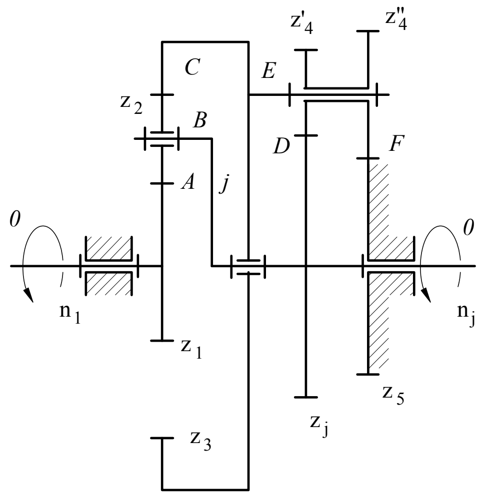


Fig. 1 A functional scheme of an selected planetary gear

Low value of ratio could be achieved in another way, however, a gear is chosen just to show a course of ratio calculations. The assumed numbers of teeth are also an example. The values are as follows: $z_1 = 20$, $z_2 = 60$, $z_3 = 140$, (-140 for Willis approach), $z_{4'} = 30$, $z_{4''} = 50$, $z_5 = 20$. Planets 2 and 4 are slightly different because planet *4* consists of two geared wheels mounted stiffly on one axis. The geared wheel *5* is braked, therefore:

$$\omega_5 = 0. \qquad (1)$$

The input element is the wheel 1 and the output element is the arm (carrier) *j*, respectively.

2 Rules of assignment: "graph ↔ planetary gear"

General rules of assignment of a graph to a gear are as follows: (i) abstraction i.e. we consider only the elements which are involved in kinematic movements of a gear (geared wheels, sun wheels, wheels with internal teeth rings, planetary gears and arms) and a support system embedded in a housing; (ii) we neglect other gear parts (e.g. bearings, seals, covers etc.) and other aspects such as e.g. friction, lubrication or vibrations; (iii) we represent the gear parts as graph vertices; (iv) we represent mutual relations of these parts as gear edges or arcs.

A linear graph of the planetary gear (*Fig.1*) is shown in *Fig. 2*. The dashed line(the edges) representspairs of two geared wheels in mesh e.g. edge (1,2) connects vertices 1 and 2, where vertex 1represents input wheel 1 and vertex 2 represents planetary geared wheel 2. Continuous line-edge *(*2,*j)* represents a pair: planet 2 and carrier *j*. The dashed-line edge *(*4‘/4“, 5) is drawn as a double line to visualize the fact that the wheel *5* is braked. The polygon *(*1, 3, *j,* 5, 0*)* is in fact a clique induced on the mentioned vertices but only for visual reasons it is usually drawn as a shaded (even) polygon according to Hsu’s idea. Exceptionally, in our case, some edges are explicitly drawn to enable a detailed explanation (i.e. edge *(*1,*j)* and edge *(*0,*j)*).

A contour graph of the same planetary gear is presented in *Fig. 3*. The original drawing rules were here preserved i.e. vertices as small dots for linear graphs and vertices as circles with vertices numbers inside them for contour graphs (Marghitu‘s), respectively.

In the case of a contour digraph (*Fig. 3*) – we draw an arc *(i,j)* if the element *i* passes rotational movement onto the element *j* or rotates around another element. E.g.: arc *(*0,1*)* symbolizes that element 1 rotates supported by a bearing system 0, arc (1,2*)* symbolizes that rotational movement is passed from wheel 1 onto planet 2, etc. Every contour has its circulation. Due to counter circulations two arcs can be drawn i.e. *(*0,3) and (3,0), simultaneously.

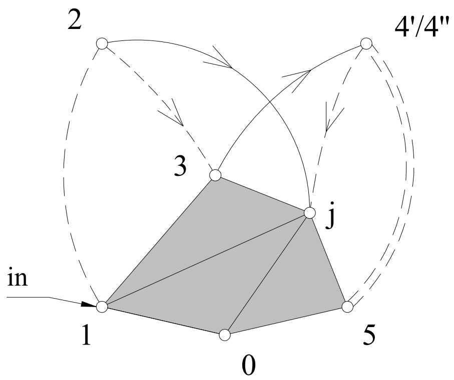


Fig. 2 Linear (Hsu's) graph of the selected planetary gear

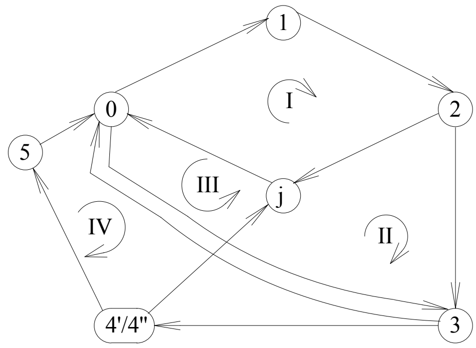


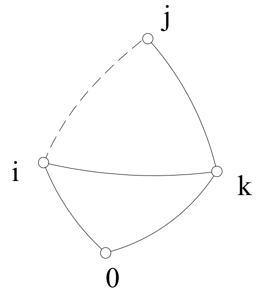
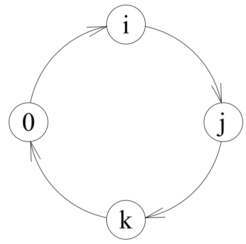
Fig. 3 Contour (Marghitu's) graph of the selected planetary gear

We distinguish sets of so called *f*-cycles [8,10] or contours in adequate graphs. Then we assign a code to each of them. Based on these codes, we can write the kinematic equations in the algorithmic manner. The selected*f*-cycle is marked by means of bold lines in Fig. 2. The selected*f*-cycle and contour were drawn by means of bold lines: (1,2)*j* and (0,1,2,*j*,0) – so there is an equivalence between these two approaches if we additionally consider the vertex *0*.

3 Graph-based kinematical analysis of planetary gears

In what follows, we would like to show how the gear ratio can be calculated. The general rule is defined: based upon the graph model of a gear - we generate a system of equations which describe the kinematics of a gear. Based upon these systems we can calculate a ratio. The rules of assignment of some types of equations are gathered in *Tab. 1*.

Tab. 1 Kinematics equation of a planetary gear for both its graph-based models

	Graph representation	
	linear graph	contour graph
Schematic representation of an elementary subgear		
Code assigned to graph representation	$(i, j)k$ or *(0,i,j,k,0)* (see *Fig. 2)*	$0 \rightarrow i \rightarrow j \rightarrow k \rightarrow 0$ (see *Fig. 3*)
Kinematics equations generated upon the codes	$\omega_i - \omega_k = \pm N_{ji}(\omega_j - \omega_k)$	$\begin{cases} \vec{\omega}_{io} + \vec{\omega}_{ji} + \vec{\omega}_{kj} + \vec{\omega}_{0k} = 0 \\ \vec{r}_A \times \vec{\omega}_{ji} + \vec{r}_B \times \vec{\omega}_{kj} = 0 \end{cases}$
	where: *i* and *j* matching (geared) elements, *k* – carrier; ω with single index – absolute rotational velocities; ω with double indices – relative rotational velocities; sign – or + depends on external or internal matching; *r* – radiuses (see *Fig. 3*); radiuses equal to 0 are omitted.	

In case of a linear graph of a gear we consider all f-cycles built upon all dashed line-edges. In case of a contour graph we consider all its contours. From the graph-theoretical point of view a contour graph consists of all independent (in the light of linear spaces theory) contours.

A code of an f-cycle (i,j)k consists of a code of dashed line-edge (i,j) where one element means a planet wheel and the second one a sun wheel or a wheel with an internal teeth ring. Description k represents a carrier of the mentioned planetary wheel. The elements are written according to the sequence of their notions – assuming that the numbers are placed before letters. The order of indices in the formulas given in *Tab. 1* is fully fixed so it reduces a possibility of mistakes.

A code of a contour can be considered in different ways, however we prefer to consider the contours, which start and end in vertex *0*. It is connected with the whole theory of contour graphs [5]. Mainly, it is due to the fact that relative quantities are considered. These quantities have to be converted into the absolute ones (9). The solution of the system of equations is obtained via eliminating all relative quantities (e.g. rotational velocities) one by one.

4 Ratio calculation

The ratio of the considered gear is calculated via three different approaches.

4.1 Willis method

In case of Willis method we can consider separately consecutive subgears and their local ratios. For the subgear consisting of elements: 1,2, 3 and j we have:

$$i_{1,3} = \frac{\omega_1 - \omega_j}{\omega_3 - \omega_j} = -\frac{z_3}{z_2} \cdot \frac{z_2}{z_1}. \tag{2}$$

For the second subgear, we can write:

$$i_{j,5} = \frac{\omega_1 - \omega_3}{\omega_5 - \omega_3} = \frac{z_5}{z_{4''}} \cdot \frac{z_{4'}}{z_j} \tag{3}$$

and finally the overall ratio can be calculated:

$$i_{1,j} = \frac{\omega_1}{\omega_j} = 1 - \frac{z_3}{z_1} \cdot \frac{z_5 z_{4'}}{z_{4''} z_j - z_5 z_{4'}}. \tag{4}$$

So, inserting the assumed teeth numbers the considered ratio is as follows:

$$i_{1,j} = 1 - \frac{140}{20} \cdot \frac{20 \cdot 30}{50 \cdot 40 - 20 \cdot 30} = -2. \tag{4a}$$

For comparison, we repeat these calculations based upon the discussed graphs (*Fig. 2* and *3*).

4.2 Linear graph approach to planetary ratio calculation

Based upon the graph (*Fig. 2*) of the considered gear, we can distinguish the following *f*-cycles: (1,2)*j*; (2,3)*j*; (4'/4'', *j*)3 and (4'/4'', *j*)3. Assigning an equation to every *f*-cycle and taking into account the assumption, we obtain the following system of equations:

$$\begin{cases} \omega_1 - \omega_j = -N_{21}(\omega_2 - \omega_j) \\ \omega_2 - \omega_j = +N_{32}(\omega_3 - \omega_j) \\ \omega_{4''} - \omega_3 = -N_{54''}(\omega_5 - \omega_3). \\ \omega_{4'} - \omega_3 = -N_{j4'}(\omega_j - \omega_3) \\ \omega_5 = 0 \end{cases} \tag{5}$$

Solving the system – we can write:

$$\omega_1 = -N_{21}\left[\left(-N_{32}\frac{N_{j4'}\omega_j}{N_{54''} - N_{j4'}}\right) - N_{32}\omega_j + \omega_j\right] + N_{21}\omega_j + \omega_j \tag{6}$$

and finally we have:

$$n_{1,j} = \frac{\omega_1}{\omega_j} = -\frac{60}{20}\left[\left(-\frac{140}{60}\frac{\frac{40}{30}}{\frac{20}{50}-\frac{40}{30}}\right)-\frac{140}{60}+1\right]+\frac{60}{20}+1=-2. \tag{7}$$

Despite the fact that the formula is slightly different from the previous one, the same ratio was obtained.

4.3 Contour graph approach to planetary ratio calculation

The course of calculations is similar. We have to write down all the contours: *(*0,1,2,*j*,0);(0,1,2,3,0);*(*0,3,4,*j*,0*)* and *(*0,3,4,5,0*)*. Every contour generates two equations here, but in general [5], also the equations concerning accelerations and forces can be written. Radiuses are measured from the main gear axis till adequate points *A, B* etc. Based upon these contours (*Fig. 3*) the system of equations can be written:

$$\begin{cases} \omega_{10}+\omega_{21}+\omega_{j2}+\omega_{0j}=0 \\ r_A\omega_{21}+r_B\omega_{j2}=0 \\ \omega_{10}+\omega_{21}+\omega_{32}+\omega_{03}=0 \\ r_A\omega_{21}+r_C\omega_{32}=0 \\ \omega_{30}+\omega_{43}+\omega_{j4}+\omega_{0j}=0. \\ r_E\omega_{43}+r_D\omega_{j4}=0 \\ \omega_{30}+\omega_{43}+\omega_{54}+\omega_{05}=0 \\ r_E\omega_{43}+r_F\omega_{54}=0 \\ \omega_{50}=\omega_5=0 \end{cases} \tag{8}$$

To solve this system, we utilize the following properties:

$$\vec{\omega}_{ij}=-\vec{\omega}_{ji} \qquad\qquad \omega_{k0}=\omega_k. \tag{9}$$

Due to the fact that all considered rotational velocities as vectors act along the same axis and angles between every r and ω are equal to 90º (sin 90º = 1) therefore we can consider scalar quantities instead of vector ones.
So eliminating all relative rotational velocities and turning some of them into the absolute ones – we have:

$$\begin{cases} \frac{r_B}{r_A-r_B}\cdot\left(\omega_1-\omega_j\right)-\frac{r_C}{r_A-r_C}\omega_1=\frac{-r_C}{r_A-r_C}\omega_3 \\ \left(\frac{r_F}{r_E-r_F}-\frac{r_D}{r_E-r_D}\right)\omega_3=\frac{-r_D}{r_E-r_D}\omega_j \end{cases}. \tag{10}$$

Additionally, we assume that all geared wheels are cylindrical and they have the same module. So, the solution of this system is the same as the previous one:

$$\frac{\omega_1}{\omega_j}=-2. \tag{11}$$

To sum up, the same final result was obtained in all three courses of calculations.

4.4 Bondgraph model of planetary gear

Bondgraphs have also been frequently used for modeling of different mechanic and mechatronic systems, but especially gears were considered very rarely. Here we consider other very simple planetary gears shown in *Fig. 4a* and *Fig. 5*. The aim is also to show a course of modeling.

The dynamical model of the considered planetary gear can be derived using a bond-graph method. The bondgraphs or power flow graphs are suitable for dynamic modeling of mechanical systems. For the single stage gear the bondgraph has the form shown in *Fig. 4b*. To draw the bongraphs, we start from so called "skeleton" (0-nodes and 1-nodes and power flow bonds) connected with ideal transformer TF. Transformer coefficient n is equal to the gear transmission ratio. The inertial elements I describe inertial forces from the pinion and the gear wheel. For the planetary gear shown in *Fig. 5*, the modeling procedure is more complicated. The skeleton has to reflect the kinematics dependences of the pinion, carrier and planetary wheels. The bondgraph shown in Fig.6 is relevant to an operation mode when the outer wheel I_3 is braked and the output power is carrier I_j.The transformer coefficient describes transition ratios. There is the commercial software that can handle the bondgraph models (i.e. Controllab 20-sim).

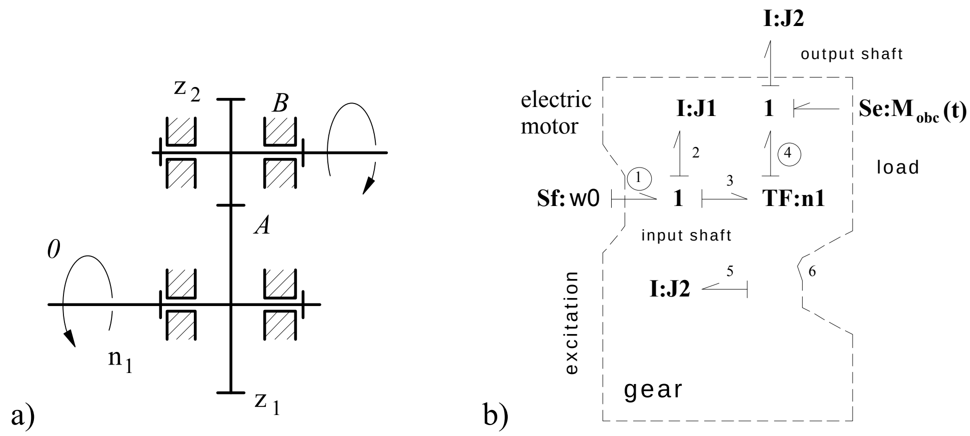


Fig.4 *Single stage spur gear and its bondgraph*

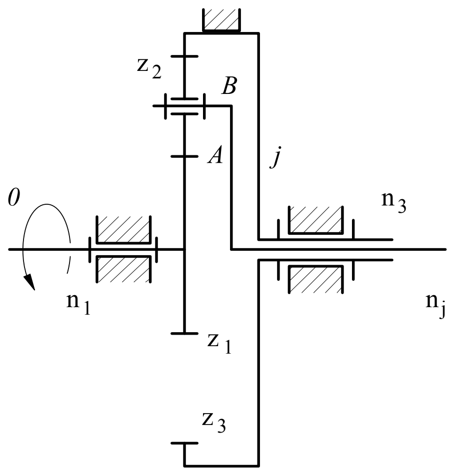


Fig. 5 *Second planetary gear*

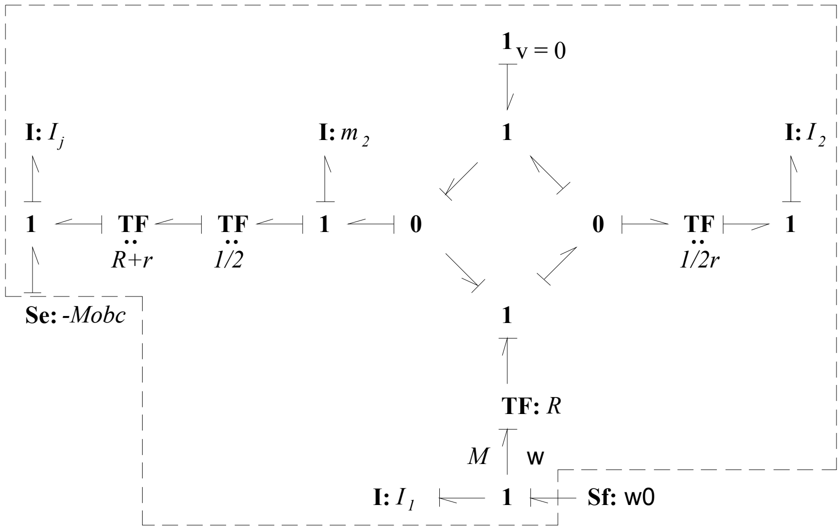


Fig. 6 Expanded bondgraph for the planetary gear (Fig. 5)

In all the figures braking elements are shown via a pictogram of brake - crossed areas. The standard software (e.g. [11]) allows for the input of bond graphs structures as well as the simulation of behavior of the modeled artifacts – because the appropriate systems of equations are generated automatically inside the system.

Conclusion

In this paper, three graph-based models were considered. The planetary gear ratio was calculated by means of Willis method as well as two graph-based methods. The identical results were obtained taking into account numerical values however the different formulas were obtained depending on the system of the notions considered in a particular model. It is a useful feature – it allows not only for checking the correctness of all approaches but also gives a deeper insight into the considered structure of a gear. The analysis of radiuses - in the case of the contour graph approach - forces a designer to check unambiguity of radiuses (especially r_E). Some further advantages of the graph-based approaches to modeling of planetary gears were shown in the cited references [4, 6, 8]. It is worth to underline that some of them e.g. creation of the complete atlases of the functional schemes of particular types of gears [8] could not be done systematically using other methods.

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ANALÝZA PLANETOVÝCH PŘEVODŮ POMOCÍ GRAFICKÝCH METOD

V předloženém příspěvku jsou diskutovány grafické modely planetových převodů založené na metodách lineárních (Hsu) grafů, křivkových (Marghitu) grafů a vazebních grafů. Původní pravidla využití grafických metod byla autory modifikována a připravena k větší využitelnosti. Byl proveden rozbor vybraného planetového převodu a představen výpočtový postup, který byl porovnán s výsledky získanými tradiční Willisovou metodou. Výhody grafických metod spočívají ve vytvoření soustavy algebraických rovnic převodu, který umožňuje aplikovat některé přístupy umělé inteligence (AI). Byl uveden model vazebního grafu dílčího převodu. Tento model umožňuje nejen vytvořit kinematické vazby, ale také řešit silové a výkonové poměry.

GRAPHEN-BASIERTE ANALYSE VON PLANETENGETRIEBEN

In der vorliegenden Arbeit werden die graphen-basierten Modelle von Planetengetrieben diskutiert. Die betrachteten Verfahren sind folgende: lineare Graphen (Hsu's), Konturengraphen (Marghitu's) und Bondgraphen. Die original zugeordneten Grafik-Regeln wurden von den Autoren angepasst und anwendungsbereiter gemacht. Ein beispielhaftes Planetengetriebe wird analysiert, um den Berechnungsgang zu zeigen für die vorgeschlagenen Methoden sowie durch die traditionelle Willis-Methode (zum Vergleich). Vorteile der graphenbasierten Methoden sind die Erstellung einer algebraischen Struktur eines Getriebes – welche eine Anwendung einiger AI-Ansätze ermöglichen. Bondgraph-Modelle eines Subsystems werden vorgestellt. Es erlaubt zusätzlich zu den algorithmischen Ansätzen nicht nur kinematische Gleichungen, sondern auch solche für die Beziehungen von Kräften und Beschleunigungen oder Leistung.

ANALIZA PRZEKŁADNI PLANETARNYCH ZA POMOCĄ METOD GRAFOWYCH

W niniejszej pracy omawiane są modele grafowe przekładni planetarnych. Rozważa się następujące metody: grafów linowych (Hsu), grafów konturowych (Marghitu) oraz grafów wiązań tzw. bondgrafów. Oryginalne zasady przyporządkowywania grafów zostały w pewnym zakresie zmodyfikowane przez autorów (niniejszej pracy) – co powoduje, że są one bardziej przydatne. Przykładowa przekładnia planetarna jest analizowana przede wszystkim dla przedstawienia przebiegu modelowania oraz obliczeń. Ponadto wyniki porównano z tradycyjną metodą Willisa. Zalety metod grafowych (modelowania przekładni) polegają na tym, że uzyskuje się algebraiczne zakodowanie (poprez wybrane struktury) przekładni planetarnej – co z kolei umożliwia zastosowanie wybranych metod sztucznej inteligencji (AI). Zamieszczono także bondgrafowy model wybranego podsystemu w rozważanej przekładni planetarnej. Ten model umożliwia także algorytmiczne generowanie równań kinematyki, a dodatkowo także zależności dla sił, przyspieszeń czy mocy.