MODIFICATION OF THE DOUBLE-ELLIPSOID HEAT SOURCE MODEL BY THE SIMPLEX METHOD

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Abstract

Despite the fact that fusion welding simulations have been used for over 20 years, their wider application has become evident only in the last 4 years. The reason is that there are both high requirements on quantity and quality of the input material data, and mainly high requirements on the accurate definition of the heat source model from which the non-stationary temperature field and temperature gradient are derived. For the conventional fusion welding methods simulation (except for the laser and electron beam), the double-ellipsoid heat source model is commonly used. However, a description of such a heat source model depends on many variables. The method that is presented in this paper could be used for the heat source parameters optimization. The method is based on fitting the computed weld pool shape to the measured one. It is a recurrent process using a simplex optimization method.

Introduction

Welding is a technological process in which a small melted area affects the character of the whole in a decisive manner. All processes take place under the special condition of rapidly changing temperature within a large temperature scale with a steep temperature gradient. The limits of temperature change are determined by the room temperature, on the one hand, and by the evaporation temperature of metal, on the other. It is then necessary to mathematically describe these processes through numerical simulations.

Welding simulations are solved in two basic stages. The first one is concerned with the calculation of non-stationary temperature fields, and it is connected to the calculation of quantity and distribution of the phases during the phase transformations. The second stage considers the calculation of stress and strain fields. It is not possible to run the second stage of the solution without prior temperature loading. Hence, the accuracy reached during the simulation of the temperature fields is totally fundamental for the final accuracy of the simulation.

For the calculation of temperature fields by the commercial Sysweld software, Furrier's differential formula is used. During welding simulations, the heat source is moving and therefore the temperature is both a function of coordinates and a function of time. It results as a non-stationary temperature field. The shape of the non-stationary temperature field is determined by the description of the heat source model used, as well as by other material characteristics.

For the simulation of fusion welding, the so-called double-ellipsoidal heat source model is used. The heat source (defined as heat flux density into material) is described by equations (1) and (2). Heat source location ξ is expressed by equation (3). The efficiency of the heat transfer into the parent material is given by the applied welding method. [1]

$$q(x,y,\xi)=\frac{6\cdot\sqrt{3}\cdot f_1\cdot Q}{a\cdot b\cdot c\cdot\pi\cdot\sqrt{\pi}}\cdot e^{\frac{-KXx^2}{a^2}}\cdot e^{\frac{-KYy^2}{b^2}}\cdot e^{\frac{-KZ\xi^2}{c^2}} \tag{1}$$

$$q(x,y,\xi)=\frac{6\cdot\sqrt{3}\cdot f_2\cdot Q}{a\cdot b\cdot d\cdot\pi\cdot\sqrt{\pi}}\cdot e^{\frac{-KXx^2}{a^2}}\cdot e^{\frac{-KYy^2}{b^2}}\cdot e^{\frac{-KZ\xi^2}{d^2}} \tag{2}$$

$$\xi = z_k - v(\tau - t) \tag{3}$$

$q(x,y,\xi)$	- Thermal flow density into the material	$(W\cdot m^{-3})$
Q	- Total source power $Q=U\cdot I\cdot\eta$	(W)
a, b, c, d	- Parameters of the melting area	(m)
ξ	- Source location depending on the welding time	(m)
x, y, z	- Point coordinates	(m)
f_1, f_2	- Constants which influence energy flow intensity into the material	$(-)$
τ	- Total welding time	(s)
t	- Immediate welding time	(s)
v	- Welding rate	$(m\cdot s^{-1})$
z_k	- Z axes coordinate when concluding welding	(m)

The Double-ellipsoid heat source model is used for most fusion welding methods' simulations (except laser and electron beam). From this, it stands to reason that the mathematical model description has to be modified by using particular welding methods. The modification itself consists of changing the coefficients *KX, KY, KZ* contained in the equations (1) a (2).

The changing of these coefficients in exponents of equations (1) and (2) affects the welding pool shape in the direction of a particular axis as well as the total temperature gradient. The problem is that the coefficient allocated to a particular axis is interconnected, and they affect each other. Therefore, the modification of this heat source is very difficult. It is to be shown in this paper how optimal heat source parameters are found in order to assure that during the numerical simulation the calculated welding pool corresponds as closely as possible to the welding pool measured experimentally.

1 Optimization process of finding parameters *KX*, *KY* and *KZ*

The main aim is to obtain a shape of the weld pool from numeric simulation which is in charge of experimental measure. The shape depends on the choice of parameters *KX*, *KY* and *KZ* of the double-ellipsoidal heat source model. The optimization process for finding these parameters uses the simplex method of Nelder-Mead [2] for local minimization of a scalar function of *n*-variables. The reason why we apply this optimization method is because it is an easy way how to find more coefficients in a short time. The simplex itself is a structure in dimension of parameters *KX*, *KY* and *KZ* which consists of *n*+1 vertexes and their functional values. The process of minimization consists in progressive moving of the simplex vertex with a maximal functional value until reaching the required accuracy. The maximum and the minimum are determined from particular functional values. It is necessary to define the simplex vertexes' center of gravity. The correct mathematical description of the algorithm can be found in the article [2] and [4]. By the application of the simplex method for seeking the convenient coefficients *KX*, *KY* and *KZ* for the double-ellipsoid heat source model, the new simplex values at each step determine the percentage difference between the surface of the weld pool from the numerical simulation, and the surface of the weld pool from the experiments. In the next simplex step the new coefficients *KX*, *KY* and *KZ* are determined in order to minimize the determined surface of the weld pool.

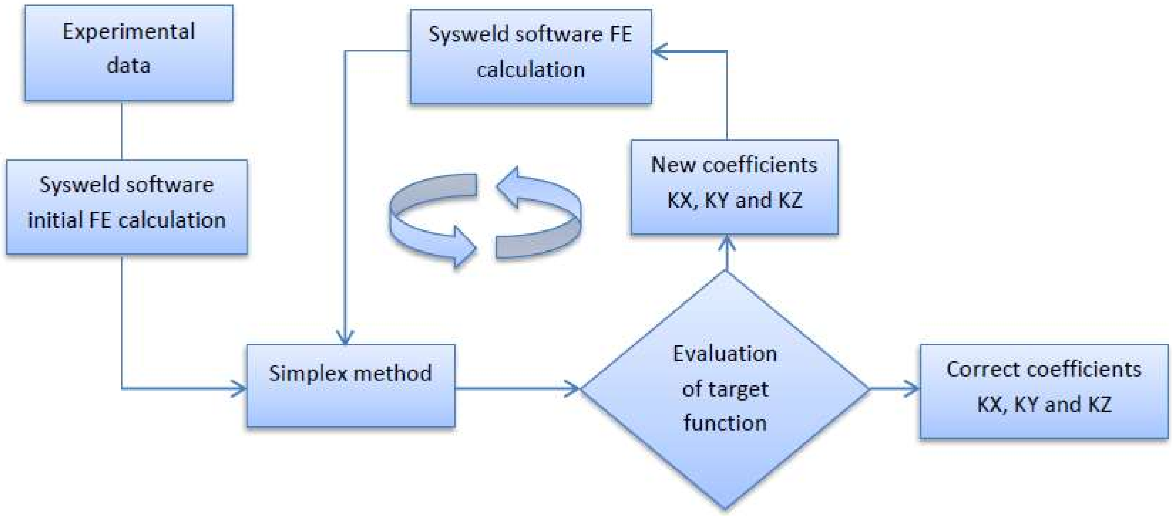

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Fig. 1: *Procedure diagram of the optimization coefficients KX, KY, KZ*

2 Procedures during the optimization of the heat source model

The aim is to find the optimal coefficients *KX*, *KY* and *KZ* in order to achieve a maximal correspondence of the weld pool shape between the simulation and experiment. These coefficients depend both on the welding method applied and also on the welding and technological parameters used within just one welding method (type of gas-shielding, type of metal transfer and so on). The equation (4) describes the match between the numerical simulation and experiment.

$$f = \sum_{i=1}^{6} w_i \left| \frac{l_{ci} - l_{mi}}{l_{mi}} \right| \tag{4}$$

In equation (4) w_i is the weight of the relevant-half axis, l_{ci} is the calculated half axis and l_{mi} is the measured distance of the half axis. Because of this mathematical description of the double-ellipsoid heat source model (equations 1 and 2), the new suggested coefficients *KX*, *KY* and *KZ* are classified. The script is written in Python (it could be used for any programming platform). The procedure diagram is shown in *Fig. 1*.

In the initial step, FE model which is based on experimentally measured data was created in software Sysweld and the double-ellipsoidal heat source model was defined. The initial coefficients *KX*, *KY* and *KZ* of this heat source are chosen by the users. After the end of the numeric simulation the temperature field is calculated (temperature in each node). This temperature field, initial proposal of coefficients *KX*, *KY* and *KZ* and experimental data are input data for optimization process (simplex method). After the running of the optimization process it is necessary to define the following data:

- The heat source coordinates
- The weights of the target function
- The material melting temperature
- Experimentally measured dimensions of the weld pool
- The maximum of the error value and number of iteration steps

The script chooses nodes lying on a straight line identical to the coordinate system with the initial point in the node corresponding with the heat source location; then they are divided into six groups corresponding to the positive and the negative axes of the coordination system. The problem of positive or negative values of target functions is solved by absolute value in equation (4). The script creates the simplex and the function values of target function are calculated in each vertexes of the simplex. The optimization process searches the highest gradient from comparison of values of the target functions in vertexes of the simplex and continues in this direction for the next calculation. If this is completed, the script will design new coefficients *KX*, *KY* and *KZ*. Then the finite element simulation is carried out again with the new parameters. This is repeated until the error of the target function is lower than written minimum error or if the difference of the calculated half axis and measured distance of the weld pool is lower than three per cent. The next condition is the number of iteration steps. If this number is bigger than the defined value, the script will be ended.

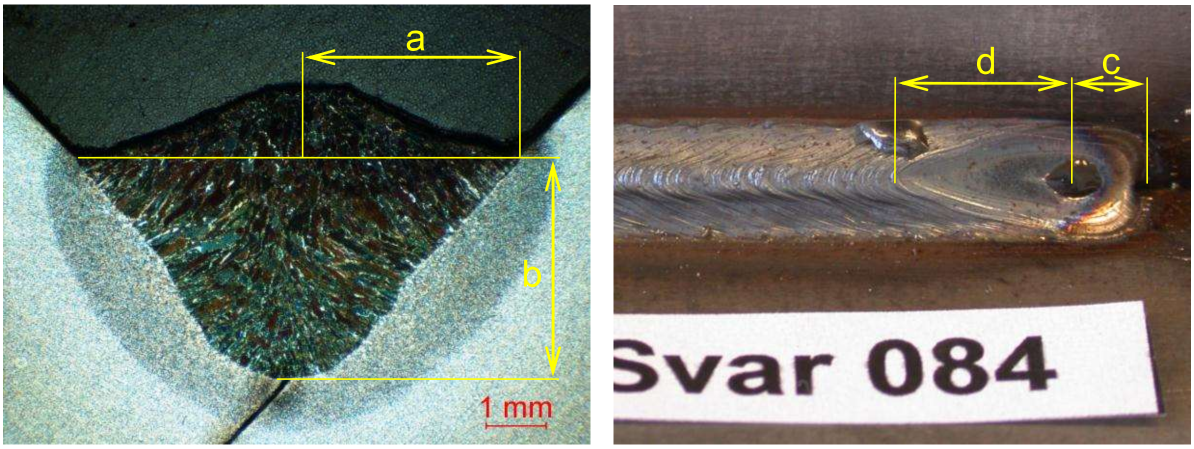

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Fig. 2: *Macro grinding and ending of the weld pool for the experimental fillet weld*

The example of the optimization parameters *KX*, *KY* and *KZ* by using the simplex method is shown on the fillet weld of material S255J2G3, which is welded by the MAG method in the shielding-gas Ar/CO_2 82/18 in *Fig. 2*. All welding parameters were monitored by the WeldMonitor system. The welding rate was 0.303 m.min^{-1}, the true welding current was 190.1 A and the true voltage was 19.7 V. The effectiveness is given by the standard (for the

MAG method; this effectiveness is 0.84 ± 0.04). The constants f_1, f_2 were chosen in a ratio of 60:40 so the constant f_1 = 1.2 and f_2 = 0.8. [3, 6]

The wire OK Autrod 12.51 with a diameter of 1.2 mm was used as a filler material. The sizes of the weld pool were found by means of metallographic scratch patterns (parameters *a*, *b*, *c*, *d* in equations 1 and 2).

Half distance of weld width	$a = 3.86$ [mm]
Maximal penetration depth	$b = 4.03$ [mm]
The first ellipsoid length	$c = 4.12$ [mm]
The second ellipsoid length	$d = 7.09$ [mm]

The result of the simulation after using all input parameters in equations (1) and (2) for the non-modified heat source model (*KX* = *KY* = *KZ* = 3) is shown in *Fig. 3*. The red colour marks the calculated fusion zone, and the blue line shows the true geometry of the weld pool. It is obvious that the calculated fusion zone is not identical with the true geometry of the weld pool.

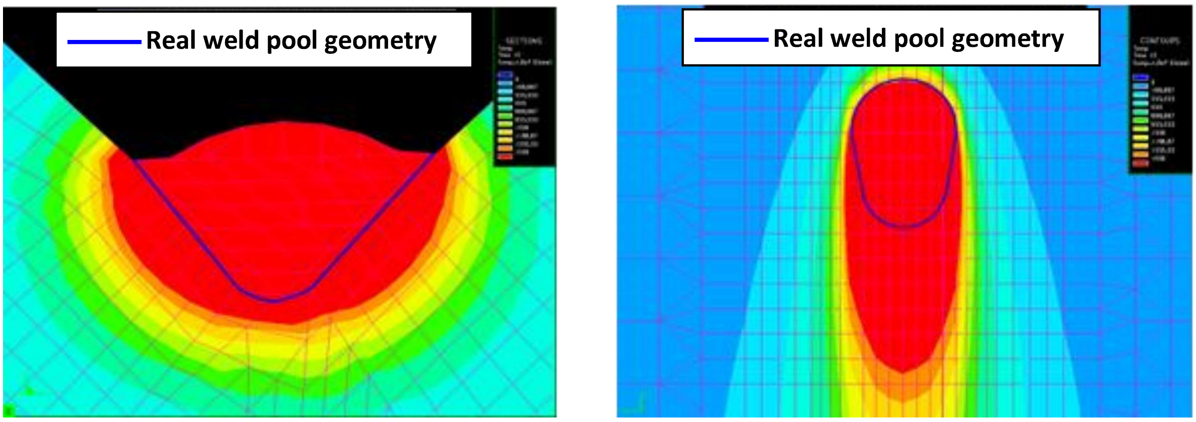

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Fig. 3: *Result of the welding simulation without a modified heat source model*

By using the described procedure based on optimization of the parameters *KX*, *KY* and *KZ* with the simplex method, the coefficients *KX* = 6.11; *KY* = 1.63 and *KZ* = 13.04 were obtained. The result of the simulation for the new modified heat source model is shown in *Fig. 4*.

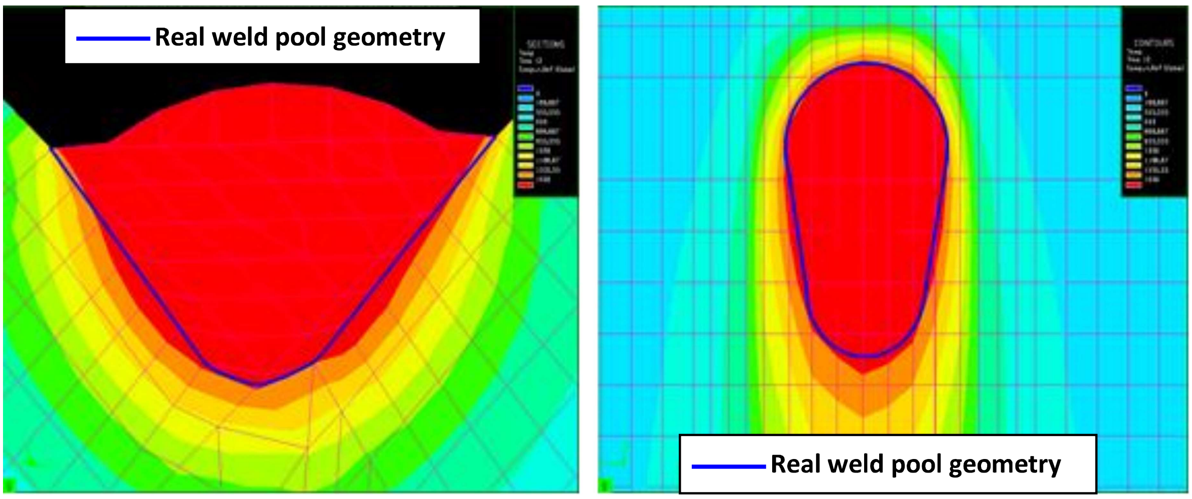

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Fig. 4: *Result of the welding simulation with a modified heat source model*

The geometry of the weld pool considerably depends on the input parameters. If the welding method is ignored, the process parameters (welding current, voltage and welding rate) and technological quantities (shielding-gas, pre-heating and metal transfer in arc) are the most significant parameters influencing geometry of the weld pool [3]. Therefore the optimal choice of the initial parameters *KX*, *KY* and *KZ* is impossible and the user uses the parameters in the form of $KX = KY = KZ = 3$.

This described method was tested on 62 welds using MAG method with different process parameters. The accuracy between numerical simulation and experimental data was 3 per cent and the maximum number of the iteration steps was 18.

Conclusion

The description in section 2 deals with the new conception of the optimization of the mathematical description of a double-ellipsoid model of the heat source while using the simplex method. It enables an immediate verification of the heat source model new mathematical description by applying the Sysweld software. It means that the mathematical description can be adjusted due to the immediate verification of the temperature fields until the required accuracy has been achieved in the direction of the particular axis. If the commercial software for welding is used, the convenient model is the taken model where the deviation in the direction of a particular axis in the weld pool is less than 8 per cent in comparison with the experimental weld pool. If the interconnection of the script and the Sysweld software is used, the deviation in the weld pool for a particular axis is less than 1 per cent both for the calculation and the experiment.

There is only one disadvantage, namely that the software using FEM is running with different script languages. For example, the main language which is used by the Sysweld software is the Systus language. The script to find out coefficients *KX*, *KY* and *KZ* is written in the Python language, which is compatible with different script languages. The big advantage of the script is the possibility of final simplex accuracy selection, on which the whole calculation time depends. The time of one calculation step depends on the number of nodes in the model. The time of one calculation step is 150 s for a model with 30,000 nodes. If the final deviation of the weld pool is less than 2 per cent, then 20 iteration steps should be sufficient to run the procedure (to obtain coefficients *KX*, *KY* and *KZ*).

Acknowledgements

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MODIFIKACE DVOUELIPSOIDNÍHO MODELU ZDROJE TEPLA SIMPLEXOVOU METODOU

Přestože simulace tavného svařování jsou využívány již více než 20 let, k jejich masovějšímu rozšíření dochází až v posledních čtyřech letech. Důvodem jsou jednak vysoké nároky na množství a kvalitu materiálových vstupních dat, ale především vysoké nároky na přesné definování modelu zdroje tepla, od něhož je odvozeno nestacionární teplotní pole a teplotní gradient. Pro simulace klasických metod tavného svařování (mimo laseru a elektronového paprsku) je využíván dvouelipsoidní model zdroje tepla. Popis tohoto modelu je však závislý na mnoha proměnných. Příspěvek si proto klade za cíl ukázat, jak lze pomocí simplexové metody a experimentů modifikovat dvouelipsoidní model zdroje tepla, aby výsledný tvar a rozměry svarové lázně odpovídaly skutečnosti, bez ohledu na použitou metodu svařování, nebo použité procesní a technologické parametry.

MODIFIKATION DES 2-ELLIPSOID-MODELLS DER WÄRMEQUELLE ANHAND DER SIMPLEX-METHODE

Obwohl numerische Simulationen des Schmelzschweißprozesses schon seit mehr als 20 Jahren angewandt werden, kam es erst in den letzten 4 Jahren zu einer allgemeinen Verbreitung. Es gibt mehrere Gründe dafür: Die Anforderung an die Menge und die Qualität der Materialcharakteristiken, aber vor allem der Anspruch an eine präzise Definition des Modells der Wärmequelle, von dem das nichtstationäre Temperaturfeld und der Gradient der Temperatur abgeleitet werden. Für die Simulationen der klassischen Methoden des Schmelzschweißprozesses (außer Laserschweißen und Elektronenstrahlschweißen) wird ein 2-Ellipsoid-Modell der Wärmequelle angewandt. Die Beschreibung dieses Modells ist von mehreren Faktoren abhängig. Dieser Beitrag hat zum Ziel zu zeigen, wie man anhand der Simplexmethode und der Experimenten das 2-Ellipsoid-Modell modifizieren kann, damit die resultierende Form und die Abmessungen des Schmelzgutes realistisch werden, unabhängig von der simulierten Schweißmethode oder den angewandten technologischen Parametern.

MODYFIKACJA MODELU PODWÓJNEJ ELIPSOIDY ŹRÓDŁA CIEPŁA METODĄ SIMPLEKS

Pomimo faktu, że symulacje procesów spawania są już znane od ponad 20 lat, do ich szerszego wykorzystania doszło w ostatnich czterech latach. Spowodowane jest to zarówno wysokimi wymaganiami co do ilości oraz jakości materiałowych danych wejściowych, ale przede wszystkim ze względu na konieczność dokładnego określenia modelu źródła ciepła, które pochodzi z niestacjonarnego pola oraz gradientu temperatury. Do konwencjonalnych symulacji metod spawania (z wyłączeniem lasera i wiązki elektronów) jest używany model podwójnej elipsoidy źródła ciepła. Opis tego modelu jest zależny od wielu zmiennych. Artykuł ma zatem na celu przedstawić, jak można przy wykorzystaniu metody simpleks i eksperymentów modyfikować model podwójnej elipsoidy źródła ciepła tak aby końcowy kształt i rozmiar jeziorka spawalniczego odpowiadał rzeczywistości, niezależnie od zastosowanej metody spawania lub użytych procedur i parametrów technologicznych.