

# FINITE ELEMENT AND BOUNDARY ELEMENT SOLUTION OF NUCLEAR WASTE REPOSITORY THERMAL DIMENSIONING PROBLEM

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## Abstract

We solve the thermal dimensioning problem of the deep geological spent nuclear fuel repository, which means to estimate the maximum temperature in the repository caused by the heat generation of the spent fuel. We use a combination of the boundary element method for the exterior problem of heat discharge to the infinity and finite element method for a near-field thermal problem with a boundary condition expressed by the far-field problem solution. This combination is implemented within the simulation software ANSYS as the “far-field element”. The far-field element solution confirmed to well represent the heat discharge, in comparison with the variant of a standard FEM far-field problem solution and conventional boundary conditions (constant temperature or zero heat flow).

**Keywords:** heat conduction, numerical simulation, far-field element, multiscale, ANSYS

## Introduction

The solved problem comes from analysis of the project of geological disposal of a spent nuclear fuel – the spent fuel is put to steel canisters which are placed deep to a stable rock massif, further protected by the buffer layer of compacted bentonite [3]. One of the issues is heat dissipation of the fuel, not to reach a certain safe maximum temperature for the construction, while keeping the repository sufficiently small for economical reasons. For numerical simulation, the challenge is in multiscale character of the problem: we need to take into account details of canister and buffer shape in scale of tens of centimeters while there are several tens or hundreds of such boreholes/canisters and the extent of thermal influence of whole repository is in scale of hundreds of meters.

The methods used in the literature [2,4,6] are e.g. a superposition of solutions of the single borehole/canister or multiscale model with line sources instead of real canister geometry, either analytically or numerically. But for changing heat power and particular canister geometry, the numerical solution is necessary anyway. Except of the analytical solution, all approaches require solution of the heat conduction problem in much larger domain than the actual volume influenced, to allow defining a realistic but simple boundary condition (e.g. undisturbed temperature). For the solution of problems in infinite domain, the boundary element method is well suited, as solution of the exterior problem. To include both correct

solution of heat conduction to infinity and particular complicated geometry of the canister, buffer, and disposal borehole, the methods can be combined together – the finite element method for the local problem and the boundary element method for the infinite (exterior) problem. Several variants are described in literature as a general concept not limited to the particular application [5].

The FEM-BEM approach is also implemented in ANSYS commercial multiphysical simulation software, in the form of the “far-field element” [1,5], i.e. the standard finite element formulation is extended with a special element attached to the boundary of the local problem, expressing the interaction of the boundary with “infinity”, equivalent to the real solution of the heat conduction in the infinite domain. Thus the problem can be solved in the much smaller computational domain and much less degrees of freedom, with all flexibility of finite elements on the local scale. In this paper we show how this approach can be efficiently used to the particular problem of thermal field of the spent nuclear fuel repository and we compare the “far-field element” solution with the solution with conventional boundary conditions.

## 1. Model description and data

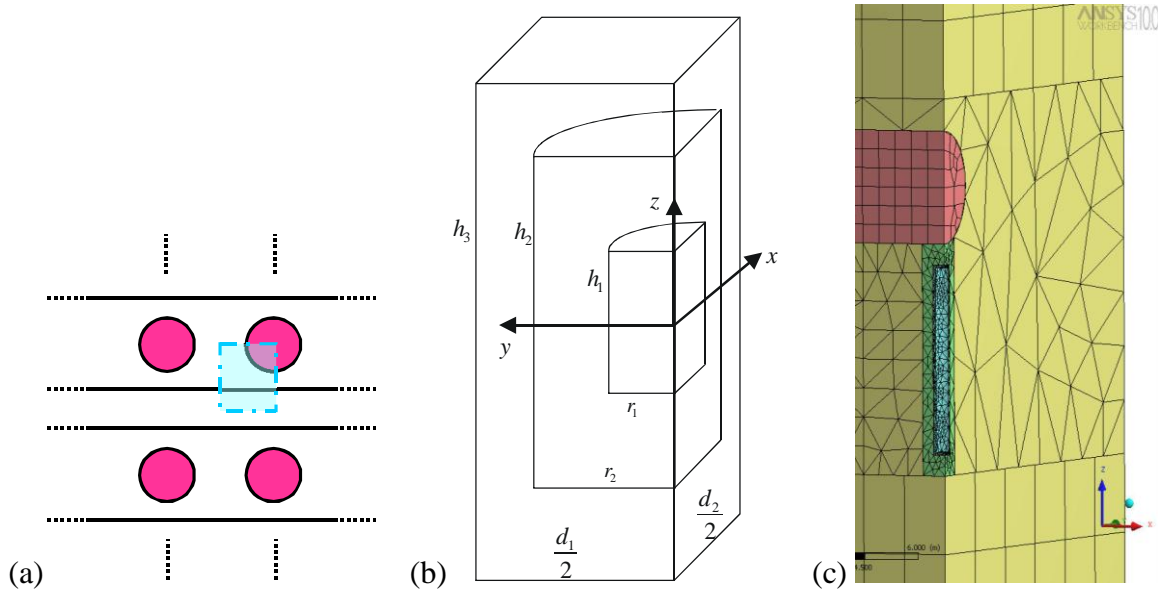
The structure of the planned repository [3,7] is a set of horizontal access tunnels with a certain spacing, with vertical disposal boreholes at the bottom of the tunnels, with a certain spacing. There is a cylindrical fuel canister in each borehole, with a layer of bentonite from top, bottom and vertical sides. We consider a periodic symmetry for the model formulation – in the plan view, the rectangle of half the tunnel spacing and half of the borehole spacing is the representative for the whole periodic structure (Fig. 1). In the vertical direction we define the domain extent according to needs of the heat influence and choice of a boundary condition. The dimensions are specified in Fig. 1b and Tab. 1.

We apply some simplifications concerning the geometry and materials. The access tunnels are filled with “backfill” material in the standard repository concept. The expected material is typically a mixture of clay and other rock, so the used thermal properties for bentonite (buffer) are not far from the possible values for the real backfill, moreover former tests have shown almost no effect of the backfill thermal properties on the buffer temperatures. We neglect the heat distribution inside the canister since the thermal dimensioning task is usually motivated by the buffer material stability, not by the phenomena inside the canister. The canister shell behaves as a perfect heat conductor distributing the heat according to the outer buffer material properties and geometry. The homogeneous volume of steel is a good approximation for the same behavior of the canister in our model. The real temperatures in the canister would be higher due to air gaps between the fuel rods and the steel skeleton/shell, but this is not a subject of this study.

**Tab. 1.** List of parameters describing model geometry (reference to Fig. 1b)

Parameter	Notations	Reference value
Borehole spacing (between axes)	$d_1$	9m
Tunnel spacing (between axes)	$d_2$	25m
Canister height (outer steel)	$h_1$	5.05m
Borehole height (outer buffer)	$h_2$	6.15m
Canister radius (steel/buffer interface)	$r_1$	0.35m
Borehole radius (buffer/rock interface)	$r_2$	0.66m
Vertical size of the model	$h_3$	60m
Tunnel diameter (circular)		3m

Source: Own compilation of data from [7,8]



Source: Own.

**Fig. 1.** (a) plan view of tunnels and boreholes (red) with symmetrical segment (blue), (b) dimensions of the problem parts, (c) discretisation mesh of the model with distinguished materials (a cut-out detail about 1/5 of the model height).

**Tab. 2.** Material coefficients of the heat conduction problem, for different components of the model (see Fig 1.c).

Material	Density	Heat capacity	Heat conductivity
	$\text{kg}\cdot\text{m}^{-3}$	$\text{kJ}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$	$\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$
Buffer (partly saturated compacted bentonite)	2000	2500	1
Host rock (granite)	2700	850	2.7
Canister (steel)	7800	45	460
Tunnel backfill = same as buffer (simplified)	2000	2500	1

Source: Own compilation of data from [8]

The input data of the model are material coefficients, boundary and initial conditions, and prescribed heat power changing in time. The heat conductivity, heat capacity and density are listed in Tab 2. The initial condition is the constant temperature  $10^{\circ}\text{C}$ . The heat power is calculated from the nuclear decay equations, for this study we got a table form of time/power dependence, possible to fit with a sum of three exponential functions [8]. The heat power at the beginning is  $2137\text{W}$ , which means the power density  $1858\text{W}/\text{m}^3$  of the homogeneous volume source mentioned above.

The boundary conditions on symmetry planes (all vertical) are no heat flow. For the boundary on the horizontal planes on top and bottom of the model, we consider the following variants (to be compared between each other):

- The “Far-field element” instead of the strict-sense boundary condition, i.e. solution of the unbounded problem.

- Constant temperature 10°C (equal to initial): this can possibly overestimate heat dissipation and underestimate temperature (keeping the temperature constant in presence of heating requires additional cooling of boundary).
- No heat flow on the bottom and the “far-field element” on the top: at the bottom side, the heat flow is underestimated and therefore the temperature is overestimated.

To resume, we solve the problem of transient 3D linear heat conduction, with either standard finite element method with mixed block and tetrahedral elements and linear or bilinear base functions, or with combination of the finite element and the boundary element method, where the solution of the exterior problem is set as a special element instead of boundary condition on particular model side. We use ANSYS v11 academic license for all calculations.

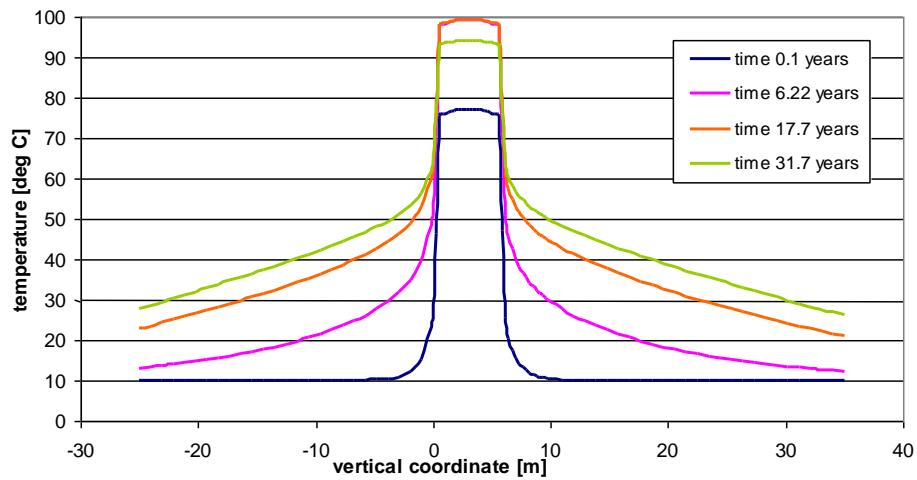
## 2. Results

We analyze the results in form of vertical temperature profiles, in order to precisely distinguish the behaviour near the boundary, which is the main subject of interest in relation to the far-field element demonstration. For the thermal dimensioning of the repository, the primary evaluated result is the maximum temperature. Surprisingly, the maximum temperature (in both space and time) is not much influenced by the choice of the boundary condition or far-field element. The reason can be that the heat power starts to decrease in similar time scale as the heat reaches the boundary (only after this time the variant of boundary can have effect on the maximum temperature). But the maximum temperature of the single profile in particular time after 10 or 20 years is then visibly influenced by the variant of the model boundary.

The profiles changing in time are presented in Fig. 2, for the far-field element variant. The quick rise of the temperature in canister and buffer and then the spread of heat towards boundaries and slower rise of the canister temperature is visible. The time development is similar to all variants of boundary. In Fig. 3 (sooner time) and Fig. 4 (later time) the profiles between model variants are compared: in the first case, the profiles are almost the same (the effect of the boundary did not yet happen) while in the second case there are differences as expected. The steepest decrease of temperature and the smallest canister temperature is for the constant temperature boundary (artificial cooling), the slowest decrease and the larger temperature is for zero heat flow (all heat kept inside), and the far-field element solution (“correct” heat conduction to the infinity) is between – the cooling is given by the infinite volume of rock around. We note that only left part of profile (bottom part of the model) is relevant, because the top of the “no heat flow” model also uses the far-field element. Also, the slope at the no-flow boundary is not ideally zero, because the model is slightly larger for this variant.

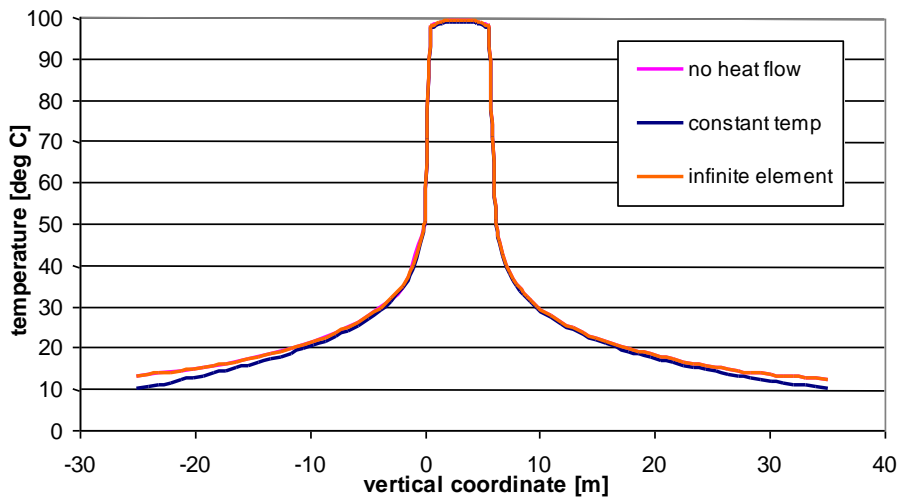
## Conclusion

The solution with the far-field element (FEM-BEM method) representing the heat conduction in the infinite domain (without need of its actual discretisation) confirmed to behave as expected. In comparison the temperature is between the results of the constant-temperature and the no-heat-flow boundary. In contrast to e.g. empirical fitting of the third-type boundary, we get the more accurate solution for almost the same computing price. The presented model is in many aspects simplified against the needs for realistic prediction of the thermal condition in the repository, but this analysis gives good reference for choice of the model geometry and boundaries in the further more precise studies.



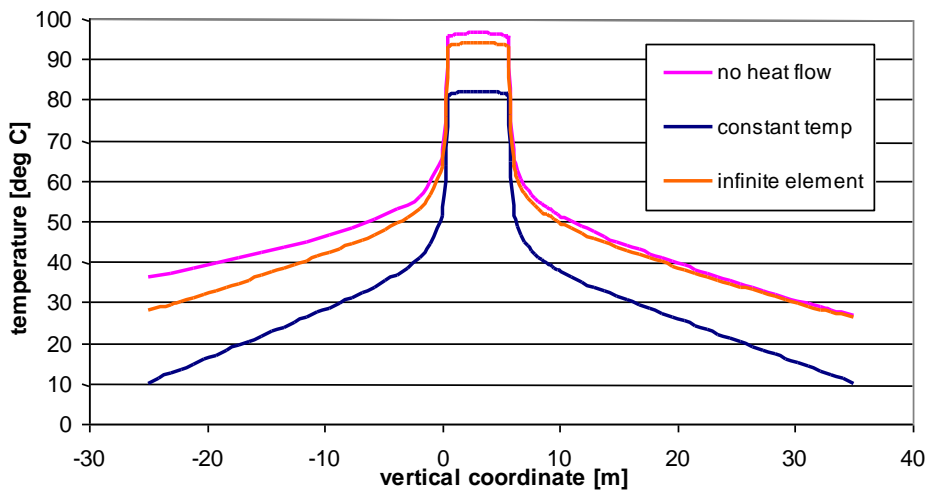
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**Fig. 1.** Development of vertical axial temperature profiles in time for the model with far-field element (infinite boundary).



Source: Own

**Fig. 2.** Comparison of temperature profiles among the model variants for a sooner time – 6.22 years (identical for “no heat flow” and “infinite element”).



Source: Own

**Fig. 3.** Comparison of temperature profiles among the model variants for later time – 31.7 years.

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## ŘEŠENÍ ÚLOHY TEPELNÉHO DIMENZOVÁNÍ ÚLOŽIŠTĚ VYHOŘELÉHO JADERNÉHO PALIVA POMOCÍ KONEČNÝCH PRVKŮ A HRANIČNÍCH PRVKŮ

Řešíme úlohu tepelného dimenzování hlubinného geologického ukládání vyhořelého jaderného paliva, což spočívá v odhadu maximální teploty v úložišti vlivem tepla generovaného uloženým palivem. Používáme kombinaci metody hraničních prvků pro vnější úlohu odvodu tepla v nekonečném prostoru a metody konečných prvků (MKP) pro tepelnou úlohu v blízkém poli s okrajovou podmínkou vyjádřenou řešením vnější úlohy. Tato kombinace je implementována v simulačním softwaru ANSYS jako "far-field element". Řešení touto metodou potvrdilo, že dobře reprezentuje odvod tepla, v porovnání s variantami modelu se standardní MKP pro úlohu vzdáleného pole nebo s volbou základních typů okrajových podmínek (konstantní teplota nebo nulový tepelný tok).

## LÖSUNG DER AUFGABE DER WÄRMEDIMENSIONIERUNG DER LAGESTÄTTE FÜR ATOMAREN ABFALL MIT HILFE FINITER ELEMENTE UND GRENZELEMENTE

Hier wird die Aufgabe der Wärmedimensionierung der Tieflagerstätte von ausgebranntem Atombrennstoff gelöst. Dies ist die Schätzung der Maximaltemperatur in der Lagerstätte, die durch den auf Grund der Wärme verbrannten Brennstoff verursacht wurde. Wir benutzen eine Kombination aus der Methode der Grenzelemente für die äußere Wärmeableitung in einen unendlichen Raum (entfernte Felder) und der Methode der finiten Elemente für die Leitung der Wärme auf dem nahen Feld mit der Randbedingung, welche durch die Lösung im entfernten Feld ausgedrückt wird. Diese Kombination wird in der Simulationssoftware ANSYS als „far-field element“ implementiert. Die Lösung unter Verwendung dieses Elements bestätigte die Fähigkeit, die Wärme korrekt abzuführen, und das im Vergleich mit der Lösung des entfernten Feldes durch finite Elemente und Standardrandbedingungen (konstante Temperatur oder kein Wärmefluss).

## ROZWIĄZANIE ZADANIA ZAGOSPODAROWANIA ENERGII CIEPLNEJ ZE SKŁADOWISKA ODPADÓW JĄDROWYCH PRZY POMOCY ELEMENTÓW SKOŃCZONYCH I ELEMENTÓW BRZEGOWYCH

W artykule przedstawiono rozwiązanie zadania zagospodarowania energii cieplnej pochodzącej z głębinowego składowiska wypalonego paliwa jądrowego, opartą na oszacowaniu maksymalnej temperatury w składowisku spowodowanej energią cieplną generowaną przez wypalone paliwo. Zastosowano połączenie metody elementów brzegowych dla zewnętrznego zadania odprowadzenia energii cieplnej w nieskończoną przestrzeń (oddalone pole) oraz metody elementów skończonych dla odprowadzenia energii cieplnej na pobliskie pole z warunkiem krańcowym wyrażonym rozwiązaniem na oddalonym polu. Połączenie to wprowadzono do oprogramowania symulacyjnego ANSYS jako "far-field element" (element oddalonego pola). Rozwiązanie z wykorzystaniem tego elementu umożliwiło prawidłową prezentację odprowadzenia energii cieplnej, w porównaniu z rozwiązaniem w postaci oddalonego pola przy wykorzystaniu elementów brzegowych oraz standardowych warunków krańcowych (stała temperatura lub zerowy przepływ energii cieplnej).