NUMERICAL SOLUTION OF THE MEW EQUATION BY THE SEMI-IMPLICIT NUMERICAL SCHEME

Jiří Hozman

Technical University of Liberec
Faculty of Science, Humanities and Education
Department of Mathematics and Didactics of Mathematics
Studentská 2, 461 17, Liberec, Czech Republic
jiri.hozman@tul.cz

Abstract

In this paper we deal with the development of a numerical method for the solution of the modified equal width wave (MEW) equation – a very important equation with a cubic nonlinearity describing a large number of physical phenomena. The crucial idea of introduced approach is based on the discretization of the MEW equation with the aid of a combination of the discontinuous Galerkin (DG) method for the space semi-discretization and the backward Euler method for the time discretization. The appended numerical experiments investigate the conservative properties of the MEW equation related to mass, momentum and energy, and illustrate the potency of this scheme, consequently.

Keywords: Discontinuous Galerkin method; modified equal width wave equation; semi-implicit scheme; solitary wave.

Introduction

Our aim is to present a sufficiently robust, accurate and efficient numerical method for the solution of scalar nonlinear partial differential equations. As a model problem, we consider a modified equal width wave (MEW) equation, describing the various phenomena in physical disciplines. The MEW equation contains a cubic nonlinearity and exhibits a pulse-like solitary wave having the same width with both positive and negative amplitudes. Several numerical methods have been introduced in the literature for the solution of the MEW equation, see [1], [6], [9] and references cited therein.

In this paper, we proposed a semi-implicit scheme for the numerical solution of the MEW equation. The discontinuous Galerkin (DG) methods have become a very popular numerical technique for the solution of nonlinear problems. The DG space semi-discretization uses higher order piecewise polynomial discontinuous approximation on arbitrary meshes, for a survey, see [2], [3], [4]. Among several variants of DG methods we prefer the so-called interior penalty Galerkin (IPG) discretizations. The discretization in the time coordinate is performed with the aid of a linearization and the backward Euler method, sidetracking the time step restriction well-known from the explicit schemes. Consequently, the fully discrete problem is represented by the system of linear algebraic equations.

The rest of the paper is organized as follows. The problem formulation and its variational reformulation are given in Section 1. The discretization including a space semi-discretization and fully time space discretization is considered in Section 2. Some numerical results are provided in Section 3.

1 Problem Formulation

Let $\Omega = (a,b) \subset I\!R$ be a bounded open interval, we consider the following MEW equation:
Problem (I): Find $u(x,t) : Q_T = \Omega \times (0,T) \to I\!R$ such that, for all $T > 0$,

$$\begin{aligned}
&\text{(a)} \qquad \frac{\partial u}{\partial t} + \varepsilon u^2 \frac{\partial u}{\partial x} - \mu \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right) = 0 \quad \text{in } Q_T,\\
&\text{(b)} \qquad u(a,t) = u_D^a(t) \text{ and } u(b,t) = u_D^b(t), \quad t \in (0,T)\\
&\text{(c)} \qquad u(x,0) = u^0(x), \quad x \in \Omega,
\end{aligned} \tag{1}$$

where positive parameters ε and μ represent the amplitude of the wave and long-wavelength, respectively. The initial boundary value problem (1) is equipped with the initial condition $u^0 : \Omega \to I\!R$ and the Dirichlet boundary conditions $u_D^a, u_D^b : (0,T) \to I\!R$ prescribed at both endpoints of the domain Ω.

In order to obtain a variational formulation of (1) we introduce the standard notation for function spaces. Let $k \geq 0$ be an integer and $p \in [1,\infty]$. We use the well-known Lebesgue and Sobolev spaces $L^p(\Omega)$, $H^k(\Omega)$, Bochner spaces $L^p(0,T;X)$ of functions defined in (0, T) with values in the Banach space X and the spaces $C^k([0,T];X)$ of k-times continuously differentiable mappings of the interval [0, T] with values in X. Further, we denote the inner product in $L^2(\Omega)$ by $(\cdot,\cdot)$, and let $H_0^1(\Omega) = \{v \in H^1(\Omega) : v(a) = v(b) = 0\}$.

Now, we are ready to introduce the following *weak formulation* of problem (1):
Problem (II): Find $u \in C^1(0,T;H^1(\Omega))$ such that, for all $t \in [0,T]$,

$$\begin{aligned}
&\text{(a)} \qquad u - u^* \in C^1(0,T;H_0^1(\Omega))\\
&\text{(b)} \qquad \frac{\mathrm{d}}{\mathrm{d}t}(u(t),v) + \varepsilon b(u(t),v) + \mu \frac{\mathrm{d}}{\mathrm{d}t} a(u(t),v) = 0 \quad \forall v \in H_0^1(\Omega)\\
&\text{(c)} \qquad u(0) = u^0 \quad \text{in } \Omega, \quad u^0 \in L^2(\Omega),
\end{aligned} \tag{2}$$

where symbol $u(t)$ stands for the function on Ω such that $u(t)(x)$, $x \in \Omega$ and the bilinear form $a(\cdot,\cdot)$ and the nonlinear form $b(\cdot,\cdot)$ are defined as

$$a(u(t),v) = \int_\Omega \frac{\partial u(t)}{\partial x} v' \,\mathrm{d}x, \qquad \text{(diffusion term)} \tag{3}$$

$$b(u(t),v) = \int_\Omega \frac{\partial f(u(t))}{\partial x} v \,\mathrm{d}x \quad \text{with } f(u) = \frac{1}{3}u^3 \quad \text{(convection term).} \tag{4}$$

Function $f(u)$ in (4) represents the physical flux.

2 Discretization

Let $\mathscr{T}_h$ ($h > 0$) be a family of the partitions of the closure $\overline{\Omega} = [a, b]$ of the domain Ω into N closed mutually disjoint subintervals $I_k = [x_{k-1}, x_k]$ with length $h_k := x_k - x_{k-1}$ and the symbol $\mathscr{J}$ stands for an index set $\{1,\ldots,N\}$. Then we call $\mathscr{T}_h = \{I_k, k \in \mathscr{J}\}$ a *triangulation* with the spatial step $h := \max_{k\in\mathscr{J}} h_k$ and the interval I_k an *element*. Let $\mathscr{E}_h = \{x_0 = a, x_1, \ldots, x_{N-1}, x_N = b\}$. Further, we label by $\mathscr{E}_h^I$ the set of all inner nodes. Obviously, $\mathscr{E}_h = \mathscr{E}_h^I \cup \{a, b\}$.

The DG method can handle different polynomial degrees over elements. Therefore, we assign a positive integer p_k as a *local polynomial degree* to each $I_k \in \mathscr{T}_h$. Then we set the vector $\mathsf{p} = \{p_k, I_k \in \mathscr{T}_h\}$. Over the triangulation $\mathscr{T}_h$ we define the finite dimensional space of discontinuous piecewise polynomial functions

$$S_{h\mathsf{p}} \equiv S_{h\mathsf{p}}(\Omega, \mathscr{T}_h) = \{v; v|_{I_k} \in P_{p_k}(I_k)\ \forall k \in \mathscr{J}\}, \tag{5}$$

where $P_{p_k}(I_k)$ denotes the space of all polynomials of degree $\leq p_k$ on I_k, $I_k \in \mathscr{T}_h$. Consequently, the approximate solution of the continuous problem (1) is sought in the space S_{hp}.

For each $x \in \mathscr{E}_h^I$ there exist two elements I_k, $I_{k+1} \in \mathscr{T}_h$ such that $I_k \cap I_{k+1} = \{x\}$. Let us denote

$$v(x^+) = \lim_{\varepsilon \to 0+} v(x+\varepsilon) \quad \text{and} \quad v(x^-) = \lim_{\varepsilon \to 0+} v(x-\varepsilon) \tag{6}$$

the *traces* of v at inner points of Ω. Moreover,

$$[v(x)] = v(x^-) - v(x^+), \quad \langle v(x) \rangle = \frac{1}{2}\left(v(x^-) + v(x^+)\right), \tag{7}$$

denote the *jump* and *mean value* of the function v at points $x \in \mathscr{E}_h^I$, respectively. By convention, we also extend the definition of the jump and mean value for endpoints of domain Ω, i.e.

$$[v(x_0)] = -v(x_0^+), \quad \langle v(x_0) \rangle = v(x_0^+), \quad [v(x_N)] = v(x_N^-), \quad \langle v(x_N) \rangle = v(x_N^-) \tag{8}$$

In case that $x \in \mathscr{E}_h$ are arguments of $v(x^-)$ or $v(x^+)$, we usually omit these arguments x^-, x^+ and write simply v^- and v^+, respectively.

2.1 Space Semi-Discrete DG Scheme

Now, we recall the space semi-discrete DG scheme presented in [8]. The crucial item of the DG formulation of model problem is the treatment of the convection part. The convection terms are approximated with the aid of the following numerical flux $H(\cdot,\cdot)$ through node $x \in \mathscr{E}_h$ in the positive direction (i.e. outer normal is equal to one):

$$H\Big(u(x^-), u(x^+)\Big) = \begin{cases} f\left(u(x^-)\right), & \text{if } A \geq 0 \\ f\left(u(x^+)\right), & \text{if } A < 0 \end{cases}, \quad \text{where } A = f'\left(\frac{u(x^-)+u(x^+)}{2}\right), \tag{9}$$

which is based on the concept of *upwinding*, for more details see [7].

Due to the cubic form of physical flux $f(u) = \frac{1}{3}u^3$, the derivative $f'(u) = u^2$ is nonnegative everywhere in Ω and the numerical flux H defined in (9) has simpler form:

$$H\Big(u(x^-), u(x^+)\Big) = f\left(u(x^-)\right), \quad x \in \mathscr{E}_h \tag{10}$$

The choice of $u(x^-), u(x^+)$ for boundary points $\{a, b\}$ is necessary to specify. Here we use:

$$u(x_0^-) = u(a^-) = u_D^a \quad \text{and} \quad u(x_N^+) = u(b^+) = u_D^b. \tag{11}$$

A particular attention should be also paid to the treatment of the diffusion terms, which include artificially added *stabilization* in order to guarantee the stability of the resulting numerical scheme. Furthermore, in order to replace the inter-element discontinuities, the semi-discrete scheme is completed with *penalty* vanishing for the continuous solution.

Therefore, we can define the *semi-discrete solution* u_h of the problem (1).

Problem (III): Find $u_h \in C^1(0,T;S_{hp})$ such that, for all $t \in [0,T]$,

$$\text{(a)} \quad \frac{\mathrm{d}}{\mathrm{d}t}\Big\{(u_h(t),v_h) + \mu a_h^\Theta(u_h(t),v_h) + \mu J_h^\sigma(u_h(t),v_h)\Big\} + \varepsilon\, b_h(u_h(t),v_h) = 0 \quad \forall v_h \in S_{hp}, \tag{12}$$

$$\text{(b)} \quad (u_h(0),v_h) = (u^0,v_h) \; \forall v_h \in S_{hp}$$

where

$$a_h^{\Theta}(u(t),v) = \sum_{k\in\mathscr{J}} \int_{I_k} \frac{\partial u(t)}{\partial x} \cdot v' \,\mathrm{d}x - \sum_{x\in\mathscr{E}_h} \left\langle \frac{\partial u(t)}{\partial x} \right\rangle [v] + \Theta \sum_{x\in\mathscr{E}_h^I} \langle v' \rangle \, [u(t)], \tag{13}$$

$$+\Theta v'(x_0^+) \cdot \left(u_D^a(t) - u(x_0^+,t)\right) + \Theta v'(x_N^-) \cdot \left(u(x_N^-,t) - u_D^b(t)\right)$$

(diffusion form)

$$b_h(u(t),v) = -\sum_{k\in\mathscr{J}} \int_{I_k} f(u(t)) \cdot v' \,\mathrm{d}x + \sum_{x\in\mathscr{E}_h^I} f(u^-(t)) \, [v] \tag{14}$$

$$- f(u_D^a(t)) \cdot v(x_0^+) + f(u(x_N^-,t)) \cdot v(x_N^-)$$

(convection form)

$$J_h^{\sigma}(u(t),v) = \sum_{x\in\mathscr{E}_h^I} \sigma[u(t)] \, [v] + \sigma(x_0) \cdot \left(u_D^a(t) - u(x_0^+,t)\right) \cdot v(x_0^+) \tag{15}$$

$$+\sigma(x_N) \cdot \left(u(x_N^-,t) - u_D^b(t)\right) \cdot v(x_N^-)$$

(penalty form)

According to value of the parameter Θ, we speak of *symmetric* ($\Theta = -1$), *incomplete* ($\Theta = 0$) or *nonsymmetric* ($\Theta = 1$) variants of stabilization of the DG method, i.e., we generally consider three variants of the diffusion form a_h^{Θ}. The penalty parameter function $\sigma : \mathscr{E}_h \to I\!R$ in (15) is defined in spirit of [5] as

$$\sigma(x) = \frac{C_W}{d(x)} \quad \text{with} \quad d(x) = \begin{cases} h_1/{p_1}^2 & , x = a, \\ \min\left(h_k/p_k^2, h_{k+1}/p_{k+1}^2\right) & , x \in \mathscr{E}_h^I \wedge \{x\} = I_k \cap I_{k+1}, \\ h_N/{p_N}^2 & , x = b, \end{cases} \tag{16}$$

where $C_W > 0$ is a suitable constant depending on the used variant of scheme and on the degree of polynomial approximation.

In order to simplify the notation we introduce the form

$$\mathscr{A}_h^{\Theta,\mu}(u(t),v) := (u(t),v) + \mu \, a_h^{\Theta}(u(t),v) + \mu J_h^{\sigma}(u(t),v), \quad u(t), v \in S_{hp}, \; t \in (0,T), \tag{17}$$

which is bilinear due to (13) and (15). Consequently, the equation (12a) can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathscr{A}_h^{\Theta,\mu}(u_h(t),v_h) + \varepsilon \, b_h(u_h(t),v_h) = 0 \qquad \forall v_h \in S_{hp}, \forall t \in [0,T], \tag{18}$$

The problem (18) represents a system of ordinary differential equations (ODEs) for $u_h(t)$ which has to be discretized in time by a suitable method.

2.2 Fully Time-Space Discrete DG Scheme

There exists a wide range of approaches for the time discretization of ODE systems resulting from the DG semi-discretization. In practical computations, the simplest time discretization is via *explicit scheme* (e.g. Euler forward scheme and Runge-Kutta methods). These schemes suffer from a strong limitation on the time step due to a *CFL-stability condition*. However, their main advantage is easy implementation. On the other hand, in order to avoid the strong time step restriction of explicit DG schemes, it is suitable to use an *implicit* time discretization.

Let $0 = t_0 < t_1 < \cdots < t_M = T$ be a partition of the interval $[0,T]$, time steps $\tau_l \equiv t_{l+1} - t_l$, and u_h^l stands for the *approximate solution* of $u_h(t_l)$, $t_l \in [0,T]$, $l = 0,\ldots,M$. The fully discrete solution of problem (12) via an implicit approach with the *backward Euler method* is defined in following way.

Problem (IV): Find $u_h^l \in S_{hp}$, such that, for $l = 1,\dots,M$,

$$\text{(a)} \qquad \frac{1}{\tau_l}\left(\mathscr{A}_h^{\Theta,\mu}(u_h^{l+1},v_h) - \mathscr{A}_h^{\Theta,\mu}(u_h^l,v_h)\right) + \varepsilon b_h\left(u_h^{l+1},v_h\right) = 0 \quad \forall v_h \in S_{hp}, \tag{19}$$

$$\text{(b)} \qquad u_h^0 \text{ is } S_{hp} \text{ approximation of } u^0.$$

The main drawback inhibiting from the fully implicit treatment in (19a) is the nonlinearity of the convection form $b_h(\cdot,\cdot)$ which leads to a nonlinear system of algebraic equations at each time step the solution of which is rather expensive and more complicated.

Therefore, in our case, the proposed *semi-implicit* approach, sidetracking the time step restriction as well as the nonlinearity of the convection form is generally based on a suitable *linearization* of this form. The nonlinearity appearing in the convection form (14) comes from the physical flux $f(u)$ the linearization of which is crucial for the later linear treatment of the form $b_h(\cdot,\cdot)$.

We use a linearization of $f(u)$ with the aid of the Taylor expansion as

$$f(u(t+\tau)) = \frac{1}{3}u^3(t+\tau) = \frac{1}{3}u^3(t) + \frac{\partial}{\partial t}\frac{1}{3}u^3(t)\tau + O(\tau^2), \quad \tau > 0,\ t \in (0,T). \tag{20}$$

Hence, differentiation and omitting of higher order terms $O(\tau^2)$ in (20) lead to approximation

$$f(u(t+\tau)) \approx \frac{1}{3}u^3(t) + u^2(t)\frac{\partial u(t)}{\partial t}\tau \approx \frac{1}{3}u^3(t) + u^2(t)\left(u(t+\tau) - u(t)\right), \tag{21}$$

the last approximation in (21) comes from the Taylor expansion of u, i.e.

$$u(t+\tau) = u(t) + \frac{\partial u(t)}{\partial t}\tau + O(\tau^2) \approx u(t) + \frac{\partial u(t)}{\partial t}\tau, \quad \tau > 0,\ t \in (0,T). \tag{22}$$

Finally, from (21) we get

$$f(u(t+\tau)) \approx u^2(t)u(t+\tau) - \frac{2}{3}u^3(t) = u^2(t)u(t+\tau) - 2f(u(t)), \quad \tau > 0,\ t \in (0,T) \tag{23}$$

and by substituting u_h^{l+1} for $u(t+\tau)$ and u_h^l for $u(t)$ in (23), respectively

$$f(u_h^{l+1}) \approx \left(u_h^l\right)^2 u_h^{l+1} - 2f(u_h^l), \quad l = 1,\dots,M. \tag{24}$$

Next, we can proceed to the linearization of an implicit treatment of b_h as

$$b_h(u_h^{l+1},v_h) = -\sum_{k\in\mathscr{J}}\int_{I_k} f(u_h^{l+1})\cdot v_h'\,\mathrm{d}x + \sum_{x\in\mathscr{E}_h^I\cup\{x_N\}} f((u_h^{l+1})^-)\,[v_h] - f(u_D^a(t_{l+1}))\cdot v_h(x_0^+) \tag{25}$$

From (24) and (25) we find that

$$\begin{aligned} b_h(u_h^{l+1},v_h) \;=\; & \\ & \underbrace{-\sum_{k\in\mathscr{J}}\int_{I_k}(u_h^l)^2\cdot u_h^{l+1}\cdot v_h'\,\mathrm{d}x + \sum_{x\in\mathscr{E}_h^I\cup\{x_N\}}((u_h^l)^-)^2\cdot(u_h^{l+1})^-\cdot[v_h] - f(u_D^a(t_{l+1}))\cdot v_h(x_0^+)}_{=:b_{hL}(u_h^l,u_h^{l+1},v_h)} \\ & -2\underbrace{\left\{-\sum_{k\in\mathscr{J}}\int_{I_k} f(u_h^l)\cdot v_h'\,\mathrm{d}x + \sum_{x\in\mathscr{E}_h^I\cup\{x_N\}} f((u_h^l)^-)\,[v_h]\right\}}_{=:b_h^*(u_h^l,v_h)} \end{aligned} \tag{26}$$

where the form $b_{hL}(\cdot,\cdot,\cdot)$ is linear with respect to its second and third components and the form $b_h^*(\cdot,\cdot)$ is in fact the original convection form (14) with homogeneous Dirichlet boundary conditions.

Since linear parts of b_{hL} are treated implicitly and the nonlinear ones together with the form b_h^* explicitly, the semi-implict treatment preserves a linear algebraic problem at each time step. In this way we arrive at the following semi-implicit method.

Problem (V): Find $u_h^l \in S_{hp}$, such that, for $l = 1,\ldots,M$,

$$\begin{aligned}
&\text{(a)} \qquad \mathscr{A}_h^{\Theta,\mu}(u_h^{l+1}, v_h) + \tau_l \varepsilon b_{hL}\left(u_h^l, u_h^{l+1}, v_h\right) = \mathscr{A}_h^{\Theta,\mu}(u_h^l, v_h) + 2\tau_l \varepsilon b_h^*\left(u_h^l, v_h\right) \quad \forall v_h \in S_{hp},\\
&\text{(b)} \qquad u_h^0 \text{ is } S_{hp} \text{ approximation of } u^0. \qquad (27)
\end{aligned}$$

The discrete problem (27) is equivalent to a system of linear algebraic equations at each time instant $t_l \in [0,T]$. The resulting method has a high order of accuracy with respect to the space coordinates and the first order of accuracy with respect to time.

3 Numerical Experiments

In this section we present the results obtained from the semi-implicit method (27) proposed for the numerical solution of the problem (1) for a propagation of a single solitary wave. The MEW equation has three conservation quantities corresponding to mass, momentum, and energy

$$I_M(u) = \int_\Omega u\,\mathrm{d}x, \qquad I_{MM}(u) = \int_\Omega (u^2 + \mu(u')^2)\,\mathrm{d}x, \qquad I_E(u) = \int_\Omega u^4\,\mathrm{d}x, \qquad (28)$$

respectively, and will be monitored to check the conservation properties of the proposed algorithm.

Let us consider the following analytical solution of (1)

$$u(x,t) = A\,\mathrm{sech}\,(k(x - x_0 - pt)) \qquad (29)$$

which represents a single solitary wave of amplitude $A = \sqrt{6p/\varepsilon}$, where p is the velocity of the wave and $k = \sqrt{1/\mu}$. The boundary and initial conditions are extracted from the exact solution (29).

In order to compare our semi-implicit approach to the schemes given in [1] and [6] we set the parameters values $A = 0.25$, $x_0 = 30.0$, $\varepsilon = 3.0$ and $\mu = 1.0$. The run of the algorithm is carried up to time $T = 20.0$ over the problem domain $[0,80]$ with the constant mesh size $h = 0.1$ and the time step $\tau = 0.05$. The computations were performed by piecewise cubic approximations with $\Theta = 0$ (incomplete variant).

Figure 1 captures the development of approximation solutions of single solitary wave from an initial condition to different time instants. Table 1 records the invariant quantities and compares obtained results with several previous schemes given in [1] and [6]. We obtained satisfactory results and quite good agreement was already achieved for a piecewise cubic approximation with reference results from [1].

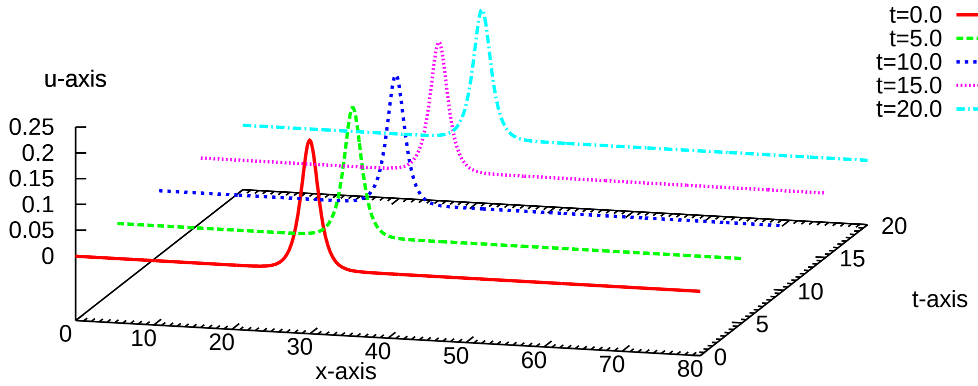
Conclusion

We dealt with the numerical solution of the MEW equation via semi-implicit scheme based on the discontinuous Galerkin method and the backward Euler method for the space and time discretization, respectively. A preliminary numerical example produced satisfactory results and illustrated the potency of the resulting scheme.

Tab. 1. Computed invariant quantities for single solitary wave

method	l time	$I_M(u_h^l)$	$I_{MM}(u_h^l)$	$I_E(u_h^l)$
present method	0.0	0.785398	0.166666	0.0052083
	5.0	0.785398	0.166666	0.0052083
	10.0	0.785398	0.166667	0.0052084
	15.0	0.785398	0.166668	0.0052084
	20.0	0.785398	0.166668	0.0052085
ref. method [6]	20.0	0.785398	0.166474	0.0052083
ref. method [1]	20.0	0.785398	0.166667	0.0052083

Source: Own based on [1] and [6]



Source: Own based on [1]

***Fig. 1.** Development of approximate solutions of single solitary wave*

Acknowledgments

This work was supported by the ESF Project No. CZ.1.07/2.3.00/09.0155 “Constitution and improvement of a team for the demanding technical computations on parallel computers at TU of Liberec”.

Literature

[1] ALI, A.; HAQ, S.; ISLAM, S.: A Numerical Meshfree Technique for the Solution of the MEW Equation. *Computer Modeling in Engineering and Sciences*, vol. 38(1), pp. 1–23, 2008.

[2] ARNOLD, D. N.; BREZZI, F.; COCKBURN, B.; MARINI, L. D.: Unified analysis of discontinuous Galerkin methods for elliptic problems. *SIAM J. Numer. Anal.*, vol. 39(5), pp. 1749–1779, 2002.

[3] COCKBURN, B.: Discontinuous Galerkin methods for convection dominated problems. In T. J. Barth and H. Deconinck, editors. *High–Order Methods for Computational Physics*,

Lecture Notes in Computational Science and Engineering 9, pp. 69–224. Springer, Berlin, 1999.

[4] COCKBURN, B., KARNIADAKIS, G. E.; SHU, C.–W., editors: *Discontinuous Galerkin Methods*. Springer, Berlin, 2000.

[5] DOLEJŠÍ, V.; HOZMAN, J.: A priori error estimates for DGFEM applied to non-stationary nonlinear convectiondiffusion equation. In G. Kreiss et. Al. Eds., *Numerical Mathematics and Advanced Applications, ENUMATH 2009*, pp. 459–468, Springer, 2010.

[6] ESEN, A.; KUTLUAY, S.: Solitary wave solutions of the modified equal width wave equation. *Comm. Nonlinear Science and Numer. Simul.*, vol. 13, pp. 1538–1546, 2008.

[7] FEISTAUER, M.; FELCMAN, J.; STRAŠKRABA, I.: *Mathematical and Computational Methods for Compressible Flow*. Oxford University Press, Oxford, 2003.

[8] RIVIÈRE, B.: *Discontinuous Galerkin Methods for Solving Elliptic and Parabolic Equations: Theory and Implementation*. Frontiers in Applied Mathematics. Society for Industrial and Applied Mathematics, Philadelphia, 2008.

[9] ZAKI, S. I.: Solitary wave interactions for the modified equal width equation. *Comput. Phys. Comm.*, vol. 126, pp. 219–231, 2000.

RNDr. Jiří Hozman, Ph.D.

NUMERICKÉ ŘEŠENÍ MEW ROVNICE PROSTŘEDNICTVÍM SEMI-IMPLICITNÍHO NUMERICKÉHO SCHÉMATU

V tomto článku se zabýváme vývojem numerické metody pro řešení MEW rovnice – velmi významné rovnice s kubickou nelinearitou popisující rozsáhlé množství fyzikálních jevů. Klíčová myšlenka uvedeného přístupu je založena na diskretizaci MEW rovnice pomocí kombinace nespojité Galerkinovy metody pro prostorovou semi-diskretizaci a zpětné Eulerovy metody pro časovou diskretizaci. Připojené numerické experimenty zkoumají vlastnosti zachování pro MEW rovnici spojené s hmotností, hybností a energií a následně tak dokládají potenci tohoto schématu.

NUMERISCHE LÖSUNG DER MEW-GLEICHUNG MITTELS EINES HALBIMPLIZITEN NUMERISCHEN SCHEMAS

In diesem Artikel beschäftigen wir uns mit der Entwicklung einer numerischen Methode zur Lösung einer MEW-Gleichung. Das ist eine sehr bedeutsame Gleichung mit einer kubischen Nichtlinearität, welche eine umfangreiche Menge physikalischer Erscheinungen beschreibt. Der Schlüsselgedanke des angeführten Ansatzes gründet sich auf der Diskretisierung einer MEW-Gleichung mit Hilfe einer Kombination aus der nichtkontinuierlichen Galerkin-Methode für die räumliche Semi-Diskretisierung und impliziten Euler-Verfahren für die zeitliche Diskretisierung. Die angeschlossenen numerischen Experimente untersuchen die Eigenschaften der Erhaltung für die MEW-Gleichung, die mit der Masse, der Beweglichkeit und der Energie verbunden ist, und belegen so die Potenz dieses Schemas.

NUMERYCZNE ROZWIĄZANIE RÓWNANIA MEW ZA POŚREDNICTWEM SEMI-DYSKRETNEGO SCHEMATU NUMERYCZNEGO

W niniejszym artykule uwagę poświęcono rozwojowi numerycznej metody do rozwiązywania równania MEW - bardzo ważnego równania nieliniowego trzeciego rzędu opisującego wiele zjawisk fizykalnych. Kluczowa idea wymienionego podejścia oparta jest na dyskretyzacji równania MEW przy pomocy połączenia nieciągłej metody Galerkina dla przestrzennej semi-dyskretyzacji oraz zwrotne metody Eulera dla dyskretyzacji czasowej. Przedstawione eksperymenty numeryczne mają na celu zbadanie cech zachowania równania MEW związanych z masą, ruchem i energią. Ponadto przedstawiaja potencjałtakiego schematu.