MEASUREMENT OF SKEWNESS OF THE ECONOMIC DATA FREQUENCY DISTRIBUTION

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Abstract

This paper is focused on the presentation and evaluation of various measures of skewness which are commonly included in statistical and economic-statistical books. It was found that they quite often provide antagonistic information and do not lead to explicit conclusions about skewness of a frequency distribution. The application of selected measures of skewness on real data was done and at the same time the simulations based on fictive data were carried out. We found that in the case when there is a classic relationship among three principal measures of central tendency $\hat{x} < \tilde{x} < \bar{x}$ $(the\ mode < the\ median < the\ arithmetic\ mean)$ or $\hat{x} > \tilde{x} > \bar{x}$, all the selected measures of skewness provide the same information about skewness of the frequency distribution. In other situations it is necessary to change the standard approach to a calculation of these measures.

Introduction

The impulse for writing this paper was the preparation of study materials and examples for the new course entitled Statistical Analysis of Data from Questionnaire Surveys which was created in the frame of the FRVŠ project Nr. 1340/2010. While creating the file of type examples the real data obtained from the questionnaire survey in the frame of the project Nr. 1/2010 with the name "Sociodemographic Survey of the Reasons of the Population Decrease in Šluknov in 2009" was used. We found that in some cases there was a problem to identify if the surveyed frequency distribution was positively or negatively skewed because selected measures of skewness took the positive as well as negative values for the same data file.

Hence, in this paper we will focus on the measures of skewness research and their practical application in empirical data analysis. The aim of our work is to evaluate the selected measures of skewness effectivity in various situations and, furthermore, to compare their information competence. The fact is that the measures of skewness fail in some conditions and data configuration and it is a serious problem while analysing data.

1 Theoretical Base of Skewness Measurement

The shape of frequency distribution research from the view of the variable values concentration plays an important role in statistical data analysis. In principle, the concentration is a cumulation of a considerable amount of values into some variant of a variable. The form of concentration can be described in two ways: in terms of a frequency curve departure from symmetry, which is called skewness, and in terms of its degree of peakedness, which is called kurtosis. In case of absence of any concentration a frequency distribution is symmetrical and at the same time rectangular, i.e. it has zero skewness and zero kurtosis.

The reason why the concentration measurement and comparison is important is the fact that we can find many situations in which the measures of central tendency as well as the measures of variation give the same information for two frequency distributions, nevertheless they are evidently different. In principle, the term skewness can be interpreted as a different concentration of small values of a variable in comparison with a concentration of large values. In positively skewed distribution (it is also called skewed to the right) there is greater concentration of small values in comparison of concentration of large values. When a distribution is negatively skewed (skewed to the left), there is greater concentration of large values in comparison with a concentration of small values there. In the case of zero skewness the frequency distribution is symmetrical. In a symmetrical distribution the mode, median and mean are equal ($\hat{x} = \tilde{x} = \bar{x}$). In a positively skewed distribution the mean is usually (but not always) the largest of the three measures of central tendency ($\hat{x} < \tilde{x} < \bar{x}$) and in a negatively skewed distribution the mode is usually the largest of the three measures of central tendency ($\hat{x} > \tilde{x} > \bar{x}$).

General requirements for measures of skewness are defined e.g. in [1, p. 306]: The measure has to assume a value of zero when the distribution is symmetrical, a positive value when the distribution is positively skewed, and a negative value in the case of negatively skewed distribution. Another important requirement is that the measure should not have any units. There is a lot of various measures of skewness which are constructed on various principles, and it is possible to find they contradict each other considerably.

A well-known measure of skewness is the measure based on extreme values – see e.g. [2, p. 44]. It compares the distance of extreme values from the median. It is a standardized statistics without any unit which can assume values between -1 and 1. Its main disadvantage is big dependence on extreme values which can occur randomly and produce a bias in results.

This disadvantage is removed by percentile measures of skewness (see [4, p. 76]) which are based on the percentile range. So, the extreme values are replaced by certain border percentiles. We would like to mention quartile measure of skewness τ_{25} first of all – see e.g. [11, p. 151].

$$\tau_{25} = \frac{\tilde{x}_{75} + \tilde{x}_{25} - 2\tilde{x}}{\tilde{x}_{75} - \tilde{x}_{25}} \qquad (1)$$

The often used measure of skewness is the moment measure α (see e.g. [1, p. 308]) which is the third moment of the standardized variable and it is possible to define it as

$$\propto = \frac{\sum_{i=1}^{k} (x_i - \bar{x})^3 n_i}{n s_x^3} \qquad (2)$$

The measure α has no units and its advantage is that it is based on all observations and it is possible to be calculated from particular subsets.

In 1895 Karl Pearson introduced the measure of skewness based on the mean, mode and standard deviation – see [8, p. 343-414]. This measure and other measures derived from Pearson's measure of skewness are often published under the different symbols, e.g. [1, p. 311]:

$$\tau' = \frac{\bar{x} - \hat{x}}{s_x} \qquad (3)$$

It is possible to find its modification based on the mean, median and standard deviation – see e.g. [1, p. 312]:

$$\tau^{''} \approx \frac{3(\bar{x}-\tilde{x})}{s_x} \tag{4}$$

This measure, after a correction, is usually defined as

$$\xi = \frac{\bar{x}-\tilde{x}}{s_x}, \tag{5}$$

see e.g. [6, p. 55]. Furthermore, V. Čermák writes that "...the measure ξ can provide disinformative values while comparing files which are a little different (from the view of skewness), so, we cannot recommend its application though it is used to being mentioned in many textbooks." – see [6, p. 55].

In 1967 L. Cyhelský created a simple measure of skewness $\tau^{''}$. He based it on the assertion that "in symmetrical distribution there is as same number of values which are greater than the mean as values which are less than the mean whereas in positively skewed distribution the number of values less than the mean predominate over the number of values greater than the mean, and in the end in negatively skewed distribution the number of values greater than the mean predominate over the number of values less than the mean" – see [3, p. 216]. This measure fills the general requirements on the measure of skewness and we can write it as

$$\tau^{''} = \frac{n^{'}-n^{''}}{n}. \tag{6}$$

V. Čermák transformed this measure as

$$\omega = \frac{n^{'}}{n} - \frac{n^{''}}{n}, \tag{7}$$

see [6, p. 56 – 59], and he called it Cyhelský´s nonparametric measure. He pointed out that this measure assumes the values between -1 and 1 for $n \to\infty$. So, it fills both the general requirements on measures of skewness and it is possible to consider it to be a standardized measure.

V. Čermák writes about this measure of skewness: "1. The measure ω is a nonparametric measure with the intent that it ignores the distance $x_i - \bar{x}$ and takes into account just their signs. 2. It is possible to apply the measure ω not only to empirical data (statistical variables) but also to random variables, especially continuous. 3. The measure ω is very sensitive to little changes of skewness in the sphere of extreme shapes of distributions." – see [6, p. 59].

The theoretical base of knowledge about skewness and its measurement is wide but it is often confined to classic situations when the distribution does not declare significant abnormality. Some of the theoretical sources mention strictly just the usual relationship of three principle measures of central tendency $\hat{x} < \tilde{x} < \bar{x}$ for positively skewed distribution and $\hat{x} > \tilde{x} > \bar{x}$ for negatively skewed distribution (see e.g. [9. pp. 54-55]). Other similar works are concerned in the relationship of the mean and median research only – see e.g. [5, p. 76]. However, this approach often fails in practice because, as Triola writes in [10, p. 92] "The mean and median cannot always be used to identify the shape of the distribution." But it is possible to say the mean and median are usually on the left side from the mode in positively skewed distribution and in negatively skewed distribution they are on the right from the mode. Though the position of the mean and median is not explicitly predictable in general, we can find situations when the position of the mode is different from the assertion mentioned above.

Graphic presentation of a distribution can sometimes be misleading, we are not often able to identify the shape of the distribution according to the graph or it is possible to make antagonistic conclusions from the graphic presentation and from the values of measures of skewness. If the distribution is positively skewed, the median is usually greater than the mode and vice versa for the negatively skewed distribution. However, sometimes the mode is equal to the median even if the distribution is not symmetrical (see [7, p. 483]).

This problem is discussed in many works on a high theoretical level; procedures are based on the probability models and are mathematically elaborated. Though, problems arise while analyzing data sets, e.g. economic data from questionnaire surveys in which we are interested in detail. We often find out that various measures of skewness give us antagonistic information and even the graphs do not help us.

2 Testing of Selected Measures of Skewness in Practice

The problem mentioned above is very serious for the practical usage of the measures of skewness because it is necessary to interpret the outcomes of our calculations and have the explicit opinion about the shape of a distribution. Therefore, we decided to apply selected measures of skewness on various frequency distributions and search how much their values are in harmony and when and why they contradict each other. We carried out the calculation of following measures of skewness on certain data sets: quartile measure τ_{25}, moment measure α, Pearson's coefficient τ', its modified version ξ and Cyhelský's measure τ''. Some distributions are also presented by graphs to be able to follow how much the graph is in harmony with conclusions resulting from the values of the measures.

If the relationship $\hat{x} < \tilde{x} < \bar{x}$ is valid in a positively skewed distribution and the relationship $\hat{x} > \tilde{x} > \bar{x}$ in a negatively skewed distribution, all selected measures of skewness give us the same information. It means they identify the shape of the distribution uniformly. The graph is in harmony with the results and has a typically presented shape.

We can detect a problem in such a situation when the mode and median are equal. The results of the selected measures are often conflicting. We will present a calculation of selected measures of skewness on real data. Let's have $\hat{x} = 3, \tilde{x} = 3, \bar{x} = 3.16$, i.e. $\hat{x} = \tilde{x} < \bar{x}$. Values of the selected measures are: $\tau_{25} = 0, \alpha = -0.027, \tau' = 0.148, \xi = 0.148, \tau'' = 0.2$. However, the graphic presentation supports the idea of a negatively skewed distribution rather than positively – see Fig. 1.

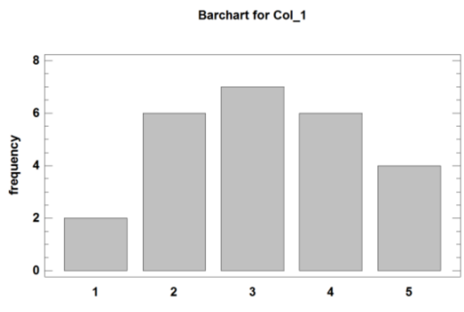

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Fig. 1: *The Example of the Frequency Distribution with the Relationship* $\hat{x} = \tilde{x} < \bar{x}$

So, the moment measure α points to negative skewness in correspondence with the graph while the other measures (τ', ξ, τ'') indicate positive skewness. Quartile measure τ_{25} which is equal to zero indicates a symmetry which is evidently not true. The reason why this measure fails in this example is the fact that the median lies exactly in the middle of the distance between the lower and upper quartile so, the numerator of this measure assumes the value of zero. The situation described above is not rare in practice. We can say the quartile measure of skewness totally and systematically fails in such a constellation of data.

We carried out other repeated simulations of the frequency distribution in which the relationship $\hat{x} = \tilde{x} < \bar{x}$ is valid and very interesting results were discovered. Firstly, it was proven that the quartile measure of skewness corresponds with the measures τ', ξ, τ'' and points to positive skewness of the distribution in the case when the median does not lie in the middle of distance between lower and upper quartile. Furthermore, we were able to simulate the situation when the relationship $\hat{x} = \tilde{x} < \bar{x}$ is valid (concretely $\hat{x} = 3, \tilde{x} = 3, \bar{x} = 3.385$) and values of the selected measures are $\tau_{25} = 1, \alpha = 0.361, \tau' = 0.417, \xi = 0.417, \tau'' = 0.231$. All values of these measures are positive and indicate that the distribution is positively skewed. This fact is also supported by a graphic presentation – see Fig. 2.

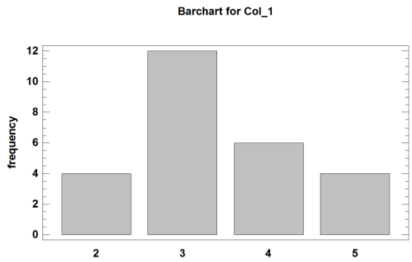

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Fig. 2: *The Example of the Frequency Distribution with the Relationship* $\hat{x} = \tilde{x} < \bar{x}$

We can say that the moment measure α usually gives different results in comparison with the other measures which we can see also in a graph.

The following example brings the same relationship between the mode and median but relation to the mean is different: $\hat{x} = \tilde{x} > \bar{x}$. We suppose that $\hat{x} = 3$, $\tilde{x} = 3, \bar{x} = 2.97$ and the values of the selected measures are: $\tau_{25} = -1, \alpha = 2.852, \tau' = -0.0295, \xi = -0.0295, \tau'' = -0.267$. This example is portrayed in Fig. 3.

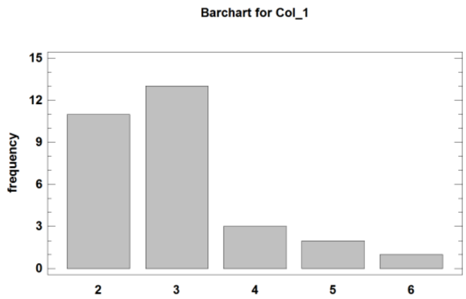

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Fig. 3: *The Example of the Frequency Distribution with the Relationship* $\hat{x} = \tilde{x} > \bar{x}$

It is apparent the value of the moment measure α points to the positive skewness and it is in harmony with the graph in Fig. 3, while the measures τ', ξ, τ_{25} and τ'' indicate negative skewness.

We also simulated the situation when the relationship mentioned above is valid and values of all the selected measures are negative. We suppose that $\hat{x} = 4, \tilde{x} = 4, \bar{x} = 3.621$ and our calculations give following results: $\tau_{25} = -1, \alpha = -0.364, \tau' = -0.394, \xi = -0.394, \tau'' =$

-0.241. These outcomes are portrayed in Fig. 4. It is possible to say the measures τ_{25}, τ', ξ and τ'' always indicates negative skewness while the measure α often gives opposed results which are also shown in a graphical visualization.

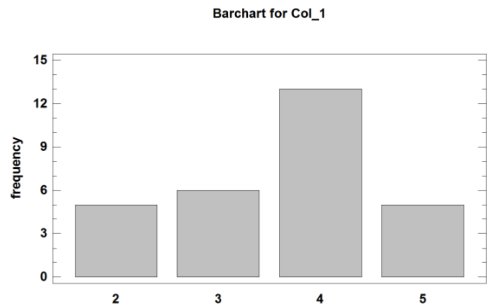

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Fig. 4: *The Example of the Frequency Distribution with the Relationship* $\hat{x} = \tilde{x} > \bar{x}$

The situation is also complicated in the frequency distributions when the position of the mean and median is opposite in comparison with the common relations. First, we will focus on the relationship $\hat{x} > \bar{x} > \tilde{x}$. We have $\hat{x} = 4, \bar{x} = 3.36, \tilde{x} = 3$ and values of the selected measures are $\tau_{25} = 0, \alpha = -0.126, \tau' = -0.551, \xi = 0.31, \tau'' = 0.04$. We can see this example in Fig. 5. It is clear the measures α and τ' indicate negative skewness with the correspondence of the graph in Fig. 5 while the measures ξ a τ'' point to positive skewness. The quartile measure τ_{25} indicates symmetry which is given by the relation of quartiles mentioned above.

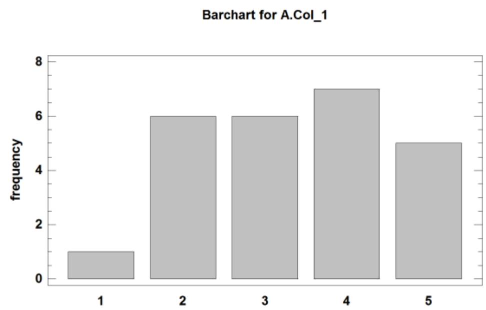

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Fig. 5: *The Example of the Frequency Distribution with the Relationship* $\hat{x} > \bar{x} > \tilde{x}$

Simulations of other distributions with a similar character and the same relationship of the three measures of central tendency showed a similar behaviour of the selected measures except the measure α. We will illustrate it by another example. Let's have the frequency distribution described by following characteristics: $\hat{x} = 5, \bar{x} = 3.154, \tilde{x} = 3$. Values of the selected measures of skewness are $\tau_{25} = 0.333, \alpha = 0.033, \tau' = -1.265, \xi = 0.106, \tau'' = 0.231$. This distribution is shown in Fig. 6. As we can see, the measure α indicates positive skewness although the structure of the three measures of central tendency is very similar to the previous example when it pointed to negative skewness. The graph in Fig. 6 shows the mode lies on the right side but other information about the shape of the distribution is rather inconsistent. The quartile measure τ_{25} indicates positive skewness too and it was proven in many following simulations. The only measure which points to negative skewness is Pearson's measure τ'. The reason is the position of the mode.

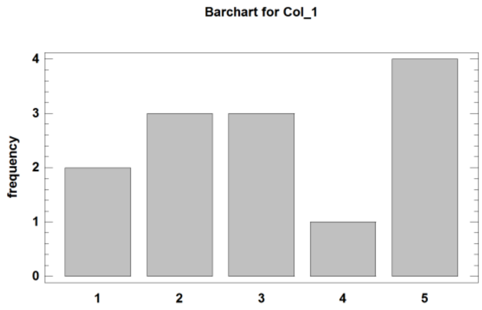

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Fig. 6: *The Example of the Frequency Distribution with the Relationship* $\hat{x} > \bar{x} > \tilde{x}$

Then, we analyse the situation when the three measures of central tendency are in the relationship $\hat{x} < \bar{x} < \tilde{x}$. Their values are $\hat{x} = 2, \bar{x} = 2.842, \tilde{x} = 3$ and the values of the measures of skewness are $\tau_{25} = -1, \alpha = 0.808, \tau' = 0.761, \xi = -0.143, \tau'' = -0.053$. This distribution is portrayed in Fig. 7. Other simulations with the similar relation of the measures of central tendency sometimes showed both positive and negative skewness. The measure τ' naturally points to positive skewness in this case.

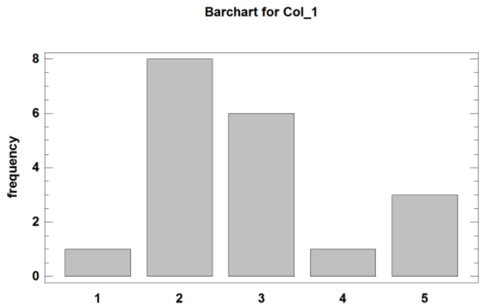

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Fig. 7: *The Example of the Frequency Distribution with the Relationship* $\hat{x} < \bar{x} < \tilde{x}$

The relation of the mode to the other measures of central tendency and its position in a frequency distribution is very questionable in general. Sometimes we cannot determine the mode. And then, it is not possible to calculate the measure τ'. This case is shown in the following example. We suppose that $\tilde{x} > \bar{x}$ and the mean equals 2.93 and the median is 3. We calculated the measures of skewness again: $\tau_{25} = 0, \alpha = 0.35, \xi = -0.052, \tau'' = -0.143$. This situation can be seen in Fig. 8. If the quartile measure τ_{25} does not misinform about symmetry of a distribution like in this case, its value is negative, i.e. in correspondence with other measures. We can find the abnormality just in the case of the moment measure α which is the only positive from all the characteristics. But we cannot say it is a rule. Sometimes when the relationship $\tilde{x} > \bar{x}$ is valid, the measure α indicates the same type of skewness as the other measures.

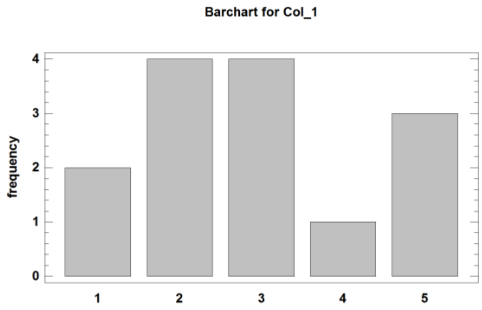

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Fig. 8: *The Example of the Frequency Distribution with the Relationship* $\hat{x} < \bar{x} < \tilde{x}$

In the situation when the median is less than the mean and we are not able to determine the mode, the measures τ_{25}, ξ, τ'' indicate the distribution is skewed to the right and the moment measure α sometimes points to positive skewness, sometimes negative. The behaviour of the measures has analogical tendencies as in the situation when $\tilde{x} > \bar{x}$.

Conclusion

The question of choosing a suitable measure of skewness and the interpretation of received outcomes is extremely complicated and we have come to the conclusion that there is not an explicit and universal solution. We suppose it is necessary to respect the specific character of surveyed data and then to choose the way of skewness measurement accordingly. Our findings about the behaviour of the selected measures of skewness are divided into two parts. Firstly, we focused on the examination of the character of a frequency distribution and we detected a few typical situations in accordance with a position of the three measures of central tendency (the mode, mean and median).

If classical relationships among the median, mode and mean – $\hat{x} < \tilde{x} < \bar{x}$ for positive skewness and $\hat{x} > \tilde{x} > \bar{x}$ for negative skewness – are valid in a distribution, all the selected measures of skewness identify positive or negative skewness in correspondence. A graph usually corresponds to the values of the statistics and has a generally presented shape – the peak of a distribution is on the left, or on the right.

A different situation arrises in such distributions where the position of the median and mode is inverse in comparison with the classic relationships. Then, information about skewness of a distribution is rather inconsistent. The measures τ', τ_{25}, ξ and τ'' indicate positive (or nagative) skewness, the moment measure α can assume both positive and negative values. The quartile measure τ' naturally assumes positive or negative value in accordance with the position of the mode.

Another problematic situation appears in the case when the mode and median are equal. In such situations the measures of skewness are often in conflict. While the measures τ', τ_{25}, ξ and τ'' indicate positive (or negative) skewness, the moment measure α gives antagonistic outcomes. The peak of the frequency distribution portrayed in a graph can be on the right, on the left or in the middle.

In the distributions where the mode cannot be determined, the measures τ_{25}, ξ, τ'' detect positive (or negative) skewness in correspondence and the measure α assumes both positive and negative values. Naturally, the measure τ' cannot be calculated.

The second aspect of our research was the evaluation of the selected measures of skewness in terms of their information competence and possibilities of their usage. In practice we can often find such situations in which the measures are not calculable or their values are evidently incorrect, i.e. the measures fail. The quartile measure τ_{25} fails in case when $\tilde{x}_{25} = \tilde{x} = \tilde{x}_{75}$ because then $\tau_{25} = \frac{0}{0}$, i.e. indeterminate expression – see [6, p. 59]. Though, a failure of this statistics is also evident in the situation when the relationship between quartiles is as follows: $\tilde{x} - \tilde{x}_{25} = \tilde{x}_{75} - \tilde{x}$ because then the measure indicates symmetry which does not exist in reality. At the same time, as we have mentioned above, this relation among quartiles is not rare.

The measure τ' which is based on comparison of the mean and mode points to symmetry of a distribution always when the mean is equal to the median. But this relation between the median and mean does not have to mean symmetry of a distribution. Moreover, the detection of mode which is necessary for the calculation can be impossible.

The measure τ'' based on comparison of number of values greater than the mean and less than the mean fails sometimes in situations when the relation $\tilde{x} > \bar{x}$ (or $\tilde{x} < \bar{x}$) is valid but the number of values greater than the mean equals the number of values less than the mean. Then, the measure indicates nonexisting symmetry.

The moment measure α relatively often indicates both positive and negative skewness in the situations which are completely the same in terms of the organisation of the mode, mean and median. This is the only measure based on all the values of a variable. It is a question if it is a positive fact. It is startling this measure usually assumes the different sign of its value in comparison with the other measures.

The results mentioned above allow us to say that each of the selected measures has its own weaknesses and limits. Therefore they fail in some situations or they are not detectable. Most of the failures consist in the fact that the measures indicate symmetry in distributions which are evidently asymmetrical, there is the only way how to correct this defect – to use other, more suitable statistics.

In the end we can say the greatest discrepancy appears while using the moment measure α which indicates in similar frequency distributions both positive and negative skewness. The measure τ' seems to be problematic too. The reason is the fact that the value of the mode is very idiosyncratic. The quartile measure τ_{25} and the measure ξ also show systematical defects but their advantage is their values are not under the direct influence of the mode so, their information competence should be better. We found the measure τ'' has the smallest defects. Its greatest advantage is completely clear and understandable interpretation, i.e. we know exactly how this measure detects skewness. While studying skewness of a distribution, it is necessary to make our expectations from the measure of skewness clear and proceed according to these expectations.

Acknowledgements

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MĚŘENÍ ŠIKMOSTI ROZDĚLENÍ ČETNOSTÍ EKONOMICKÝCH ÚDAJŮ

Tento článek je zaměřen na předložení a zhodnocení různých měr šikmosti, které se běžně uvádějí ve statistické a ekonomicko-statistické literatuře. Bylo zjištěno, že při jejich praktickém použití na konkrétních údajích dávají poměrně často protichůdné informace a nevedou k jednoznačným závěrům o šikmosti daného rozdělení. Byla provedena aplikace vybraných měr šikmosti na příkladech dat z konkrétní praxe a zároveň proběhly i simulace na smyšlených údajích. Bylo zjištěno, že v případě, kdy nastává mezi třemi nejdůležitějšími charakteristikami úrovně jeden z klasických vztahů $\hat{x} < \tilde{x} < \bar{x}$, resp. $\hat{x} > \tilde{x} > \bar{x}$, pak všechny uvedené míry šikmosti identifikují kladnou či zápornou šikmost shodně. V ostatních případech je třeba změnit standardní přístup k výpočtu těchto měr.

DAS MESSEN DER SCHIEFE DER HÄUFIGKEITSVERTEILUNGEN VON ÖKONOMISCHEN DATEN

Dieser Artikel ist auf die Beschreibung und Bewertung verschiedener Maße von Schiefen, die in der statistischen und ökonomisch-statistischen Literatur aufgeführt werden, orientiert. Es wurde festgestellt, dass die Verwendung dieser Maße mit konkreten Daten in der Praxis relativ oft zu widersprüchliche Informationen führt und nicht zu einem eindeutigen Beschluss über die Schiefe von einer bestimmten Verteilung führt. Es wurden bestimmte Schiefenmaße bei einigen Beispielen aus der Praxis berechnet. Zugleich wurden die gleichen Maße mit fiktiven Zahlen berechnet. In dem Fall, wenn unter den wichtigsten Lageparametern eine von den klassischen Relationen $\hat{x} < \tilde{x} < \bar{x}$ oder $\hat{x} > \tilde{x} > \bar{x}$ gilt, wurde festgestellt, dass alle genannten Schiefenmaße übereinstimmend auf die positive oder auf die negative Schiefe zeigen. In anderen Fällen ist es notwendig, die konventionelle Einstellung zu ändern.

POMIARY UKOŚNOŚCI ROZDZIELENI CZĘSTOŚCI DANYCH EKONOMICZNYCH

Ten artykuł koncentruje się na prezentacji i oceny różnych pomiaru ukośności, które się często pojawiają się w literaturze statystycznej oraz ekonomiczno-statystycznej. Było stwierdzono, że w praktyce na specyficznych informacjach, może dać często sprzeczne informacje i nie prowadzą do jednoznacznych wniosków na temat ukośności rozdzieleni.Była wykonana aplikacja różnych pomiaru ukośności na przykładach danych z określonej praktyki i jednocześnie wykonana symulacja na fikcyjnych danych. Było stwierdzono, że w przypadku, kiedy pomiędzy trzema najważniejszymi cechami poziomu jeden z klasycznych zależności $\hat{x} < \tilde{x} < \bar{x}$, lub $\hat{x} > \tilde{x} > \bar{x}$, później wszystkie wspomniane pomiary ukośności rozpoznają pozytywną bądz negatywną ukośność identycznie. W innych przypadkach się musi zmienić standartową metodę do obliczania tych pomiaru.