IMPLEMENTATION OF AUSM$^+$-UP SCHEME WITH IMPLICIT RESIDUAL SMOOTHING INTO OPENFOAM

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Abstract

The standard installation of open-source package OpenFOAM consists of segregated solvers based on pressure-correction schemes. These schemes specify type of jobs that can be solved, mainly incompressible and subsonic compressible flows. This paper deals with the implementation of coupled scheme with AUSM$^+$-up flux that allows solving fluid problems in all speed regimes. This scheme is complemented by the Implicit Residual Smoothing (IRS), which extends the region of stability. Therefore an increase in the speed of convergence is observed. Acceleration by factor 6 was achieved.

Introduction

There are many codes that allow solving problems of fluid mechanics. Mostly commercial codes are popular in industry because they provide adequate accuracy and stability for most of cases and they are well documented and prepared for everyday use. In-house codes are mostly opposite because they are oriented mostly on certain case and use of this code requiring more knowledge in mathematics and computer skills. There is another variant that are based on public codes where basic schemes and user interfaces are implemented and they can be easily modified. OpenFOAM is one of these packages which are mostly based on pressure-correction schemes such as SIMPLE or PISO algorithms [7]. These algorithms allow solving especially cases of incompressible or subsonic compressible flow. There is missing an algorithm that is suitable for flow from subsonic incompressible regime (0.1 Ma) to supersonic regime (e.g. 3 Ma).

The AUSM$^+$-up scheme presented by Liou [2] was chosen. This algorithm allows solving the flow in a wide range of Mach numbers. Implementation of implicit variant of AUSM$^+$-up is difficult if we are using standard resources of OpenFOAM. Therefore explicit variant of AUSM$^+$-up was chosen that is only conditional stable. This limitation of time step should lead to inefficient method especially in the case of viscous flow. This is the reason why the implementation of AUSM+up scheme was complemented by method of Implicit Residual Smoothing (IRS) [3, 4] in combination with Runge-Kutta method (RK) [3, 4, 6]. The resulting scheme has improved stability and speed of convergence.

1 Numerical schemes

1.1 AUSM$^+$-UP

Let us consider a hyperbolic system of equations described in (1) where $W = (\rho, \rho u, \rho E)^T$ is a vector of conservative variables in 1D.

$$W_t + \nabla \cdot F = 0 \tag{1}$$

The vector of fluxes F consists of convective ϕ, ψ and pressure parts P as is shown in (2).

$$F = \phi\psi + P \tag{2}$$

$$\phi = \rho u, \psi = (1, u, H)^T \text{a} \, P = (0, p, 0)^T \tag{3}$$

The numerical flux through cell boundary is then

$$f_{1/2} = \phi_{1/2}\psi_{L/R} + p_{1/2}, \tag{4}$$

where

$$\psi_{L/R} = \begin{cases} \psi_L \ if \ \phi_{1/2} \geq 0 \\ \psi_R \ if \ \phi_{1/2} < 0 \end{cases}, \tag{5}$$

and

$$\phi_{1/2} = u_{1/2}\rho_{L/R} = a_{1/2}M_{1/2}\rho_{L/R}, \phi_{L/R} = \begin{cases} \phi_L \ if \ M_{1/2} \geq 0 \\ \phi_R \ if \ M_{1/2} < 0 \end{cases} \tag{6}$$

Functions $p_{1/2}$ and $M_{1/2}$ are defined as

$$M_{1/2} = M_{(4)}^+(M_L) + M_{(4)}^-(M_R) + 2\frac{K_p}{f_a}max(1 - \sigma\bar{M}^2, 0)\frac{p_R - p_L}{(\rho_L + \rho_R) + a_{1/2}^2}, \tag{7}$$

$$p_{1/2} = P_{(5)}^+(M_L)p_L + P_{(4)}^-(M_R)p_R - K_u P_{(5)}^+ P_{(5)}^-(\rho_L + \rho_R)f_a a_{1/2}(u_R - u_L). \tag{8}$$

Definitions of the used functions, constants as well as the complete derivation is shown in [2].

1.2 Runge-Kutta

The integration in time is achieved with the Runge-Kutta [3, 4, 6] method

$$W^j = W^n, j = 0$$

$$W^{j+1} = W^n - \alpha_j \Delta t R(W^j) \tag{9}$$

$$W^{n+1} = W^j, j = m$$

Where residuum R in cell i with volume $|\Omega_i|$ is defined as

$$R_i = \frac{1}{|\Omega_i|}\sum_{j \in neighbour} F_{ij} \tag{10}$$

Where F_i is a component of vector F defined by equation (2).

It is shown e. g. in [6] that this scheme has a wider stability region than the explicit Euler scheme, although the time step is still severely limited by the stability condition $(\Delta t_{RK} = \kappa(\alpha_j)\Delta t_{Euler})$ as is shown in tables 1 and 2.

1.3 Implicit Residual Smoothing

Next improvement can be achieved with the so called Implicit Residual Smoothing (IRS) [3, 4]. The IRS method introduces a smoothing operator for residual R, i.e. instead of by eq. (11)

one solves a smoothed version by eq. (12). Stability analysis in [3] has shown that the system of equations (12) is unconditionally stable.

$$W^{n+1} = W^n - \Delta t R, \tag{11}$$

$$W^{n+1} = W^n - \Delta t \tilde{R}, \tag{12}$$

Smoothed residuum $\widetilde{R}$ is defined as

$$(1 - \epsilon \Delta)\tilde{R} = R \tag{13}$$

where ϵ is a smoothing parameter. System of equations (13) is solved by Jacobi iteration solver as is shown in (14).

$$\tilde{R} = \frac{R + \epsilon \sum_k^m R_k}{1 + \epsilon m}. \tag{14}$$

Sometimes it is suitable (e.g. in the case of computational grid with local refinement) to use residual smoothing in the following form
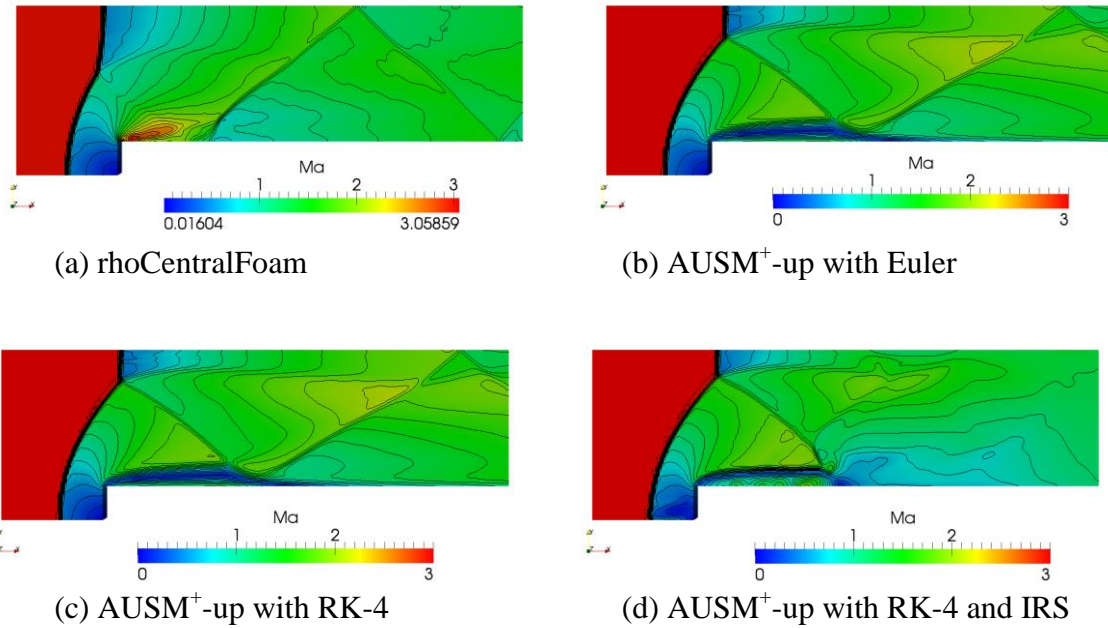
$$\tilde{R} = \frac{R + \epsilon \sum_k^m R_k \frac{\Psi_k}{\Sigma \Psi_k}}{1 + \epsilon m}. \tag{15}$$

Ψ_k is the weight defined by distance of two neighbor cells and face magnitude between these cells

$$\Psi_k = \frac{S_{ij}^2}{|x_i - x_j|}. \tag{16}$$

2   Results

The AUSM scheme with Implicit Residual Smoothing (IRS) and Runge-Kutta time marching were tested on two cases. In both cases three approximations of time derivatives are compared. The first one is a forward Euler scheme, the second one is a four-step Runge-Kutta scheme with coefficients 0.11, 0.2766, 0.5 and 1. The third one is a four-step Runge-Kutta (RK4) scheme with Implicit Residual Smoothing.

(a) rhoCentralFoam

(b) AUSM$^+$-up with Euler

(c) AUSM$^+$-up with RK-4

(d) AUSM$^+$-up with RK-4 and IRS

Source: Own

Fig. 1: *Forward facing step – Mach number*

2D forward facing step is the first test case. The computational grid consists of 16k square cells. The flow is characterized by inlet Mach number M = 3, the gas is assumed as an ideal gas with heat capacity ratio 1.4. One can observe a difference between flow fields obtained by native OpenFOAM solver rhoCentralFoam based on Kruganov-Tadmor scheme and our implementation of AUSM$^+$-up scheme as is shown in figures 1 (fields of Mach number) and 2 (pressure distribution on lower wall). All methods are consistent in the determination of front shock wave. The main difference is behind step where each algorithm predicts different structure of shock wave reflection and interaction. The change of time approximation between Euler and 4 stage Runge-Kutta scheme has no effect on flow field and pressure profile. These results are similar with results presented in [9]. If IRS is enabled the shock wave reflection from bottom wall is not captured and flow field is changed due residual averaging.

The stable time step and required time for solution of 10 s of physical time is shown in table 1. Application of Runge-Kutta method allowed increasing time step 5e-4 s ten times with reduction of computational time approximately to 1/3. If the IRS is enabled, the stable time step increases to 3e-2 s and reduces computational time approximately to 1/10 of the original computational time.
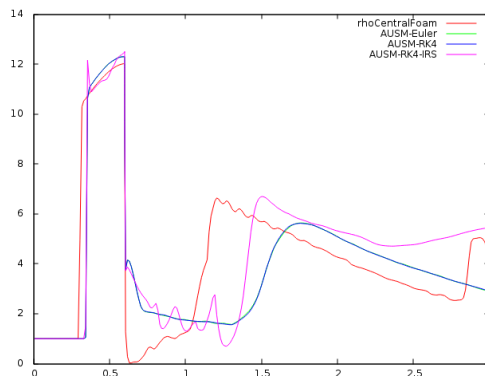
Source: Own

Fig. 2: *Pressure on bottom wall of channel*

Tab. 1: *Comparison of stable time step and solution time of 10 s physical time for different time integration settings*

	Stable time step [s]	Solution time [s]
Euler	5e-4	568
RK4	5e-3	220
RK4+IRS	3e-2	65

Source: Own

In the second case the efficiency of various approximations were compared on 2D inviscid transonic flow around turbine blade SE-1050 (Fig. 2.) The computational domain consists of 6k triangular cells and it is constructed with periodic boundary conditions to simulate effect of infinite turbine cascade. This simulation was computed with following boundary conditions given in [8]

- total pressure 980717 Pa and temperature 298.65 K on inlet

- angle of attack 70,7°

- isoentropic outlet Mach number $M_{2i}=1,19$

- thermo physical properties of the gas corresponds to air (neglecting the viscosity)

The AUSM[+]-up scheme captures two shock waves generated on trailing edge of blade and reflection of the inner branch from suction surface. It is possible to see a similarity between our and published results [8] in figure 3. AUSM[+]-up predicts the outlet Mach 1.15. This is slightly lower than assumed isoentropic Mach number 1.19 due to energy losses. Pressure distribution along blade is shown in figure 4. There is an agreement between experimental and computed pressure distribution around blade.
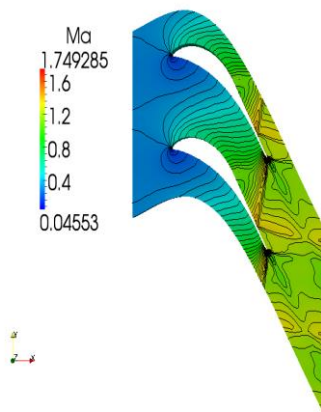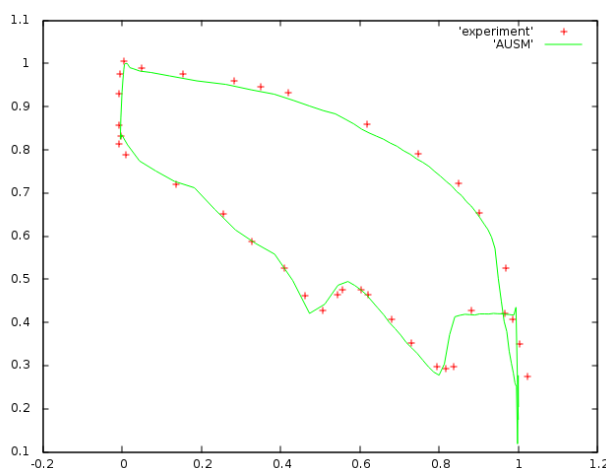

Source: Own   *Source: [8]*

Fig. 3: *Comparison between computed and published Mach number fields*

Fig. 4: *Comparison of pressure distribution along blade*

Four-step Runge-Kutta scheme with IRS allows to use 20 times higher time step and reduces the solution time approximately 6 times as is shown in table 2. Pure 4 step Runge-Kutta schemes allows to use 4 times higher time step but with similar computational cost.

Tab. 2: *Comparison of stable time step and solution time of 0.0001 physical time*

	Stable time step [s]	Solution time [s]
Euler	5e-7	210
RK4	2e-6	201
RK4+IRS	1e-5	36,6

Conclusion

AUSM$^+$-up scheme which is possible to use in wide interval of velocities was implemented Into OpenFOAM. This scheme was supplemented by Runge-Kutta scheme and Implicit Residual Smoothing for improvement stability and convergence speed. The improvement was confirmed by simulation of two inviscid compressible cases. Implemented scheme is universal with wide range of applications. This scheme was used e. g. for simulation of 3D compressible flow in turbine cascade or test facilities for internal re-circulation surge suppression in latest time. There is a potential to apply this algorithm in the case of jet engine simulation especially for simulation of combustor – turbine interaction. Therefore it would be useful to extend computational algorithm for support of dynamic mesh, mesh refinement or combustion simulation.

The IRS is useful extension that is easy to implement by standard OpenFOAM operators. Future work will be oriented into coupling AUSM$^+$-up and IRS with other OpenFOAM libraries for supporting combustion modeling, dynamic mesh refinement or motion.

Literature

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IMPLEMENTACE SCHÉMATU AUSM$^+$-UP S IMPLICITNÍM VYHLAZOVÁNÍM RESIDUÍ DO SOFTWARU OPENFOAM

Standardní instalace open-source balíku pro výpočetní mechaniku tekutin OpenFOAM je založena hlavně na schématech kombinujících tlak a rychlost. Tato schémata předurčují typy úloh, které je možné pomocí nich řešit (např. podzvukové nestlačitelné, stlačitelné nebo transsonické). Tento článek se zabývá implementací schématu AUSM$^+$-up založeného na provázání hustoty a rychlosti, které umožňuje řešit problémy mechaniky tekutin v širším intervalu rychlostí. Toto schéma je doplněno o implicitní vyhlazování residuí, které rozšiřuje oblast stability použitého schématu. Proto je pozorován nárůst rychlosti konvergence použitého schématu, které bylo zrychleno šestkrát.

IMPLEMENTATION DES AUSM$^+$-UP SCHEMAS MIT IMPLIZITER GLÄTTUNG VON RESIDUEN IN OPENFOAM SOFTWARE

Die Standartinstallation eines Open-source-Pakets OpenFOAM für die Berechnung von Flüssigkeitsströmungen basiert hauptsächlich auf Lösungsschemen, die Druck und Geschwindigkeitsfelder verknüpfen. Diese Schemen geben auf diese Weise lösbare Aufgabentypen vor (z.B. Unterschallströmung, inkompressible/ kompressible oder transsonische Strömung). Dieser Artikel setzt sich mit der Implementierung des Schemas AUSM$^+$-up auseinander, das auf der Verknüpfung von Dichte und Geschwindigkeitsfeldern beruht. Die Lösung eines breiteren Aufgabenfeldes in der Flüssigkeitsberechnung wird dadurch ermöglicht. Mit Hilfe der Glättung von Residuen wird das Schema ergänzt, damit der Stabilitätsbereich erweitert wird. Auf Grund der berechneten Aufgabe wurde die Konvergenz stark beschleunigt und eine sechsfache Geschwindigkeitserhöhung beobachtet.

WDRAŻANIE SCHEMATU AUSM$^+$-UP Z WYGŁADZANIEM REZYDUÓW W OPENFOAM SOFTWARE

Standardowa instalacja pakietu open-source do obliczeniowej mechaniki płynów OpenFOAM oparta jest przede wszystkim na schematach łączących ciśnienie i prędkość. Schematy te określają rodzaje zadań, które można z ich pomocą rozwiązać (np. poddźwiękowe nieściśliwe, ściśliwe lub transsoniczne) Niniejszy artykuł poświęcony jest wdrażaniu schematu AUSM$^+$-up opartego na powiązaniu gęstości i prędkości, które umożliwia rozwiązywanie problemów mechaniki płynów w szerszym przedziale prędkości. Schemat ten został uzupełniony wygładzaniem rezyduów, które rozszerza zakres stabilności zastosowanego schematu. Dlatego obserwowany jest wzrost prędkości konwergencji zastosowanego schematu, która została przyspieszona sześciokrotnie.